GCSE-AS Mathematics Bridging Course

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Chellaston School

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Monday Objectives

- ★ The Equation of a Line
- ★ Surds
- ★ Linear Simultaneous Equations

Tuesday Objectives

- ★ Factorising Quadratics & Equations
- ★ Non-Linear Simultaneous Equations
- ★ Graphical Intersections (Line and Curve)

Wednesday Objectives

- ★ Completing the Square
- ★ Solving Complex Equations
- ★ The Quadratic Formula

Thursday Objectives

- ★ Numerical Fractions Review
- ★ Indices
- ★ Algebraic Fractions

Friday Objectives

- ★ The Sine and Cosine Rules
- ★ Review Questions on Bridging Topics
- ★ The Area of a Triangle
- ★ Assessment

REFERENCE IN OTHER RESOURCES

Head Start: Copies in school during week / purchase details provided.

Alpha Workbooks: Copies in school during week / purchase details provided.

CONTENTS

All material written by Paul Leary.

Chellaston School, 2010.

THE EQUATION OF A LINE

The Equation of a Line

Objectives

★ find gradients of straight lines

- ☆ by drawing a triangle on a diagram
- ☆ by calculation from a pair of coordinates
- ☆ from the equation of a straight line graph

★ know that parallel lines have equal gradients

★ know the relationship between gradients of perpendicular lines

★ know and use formulae that give straight line graphs parallel to the axes

- ☆ $x = k$ (vertical line)
- ☆ $y = k$ (horizontal line)

★ know and use formulae that give diagonal straight line graphs

- ☆ $y = mx + c$ (the gradient and intercept form!)
- ☆ $ax + by = k$

★ rearrange simple formulae

★ determine the equation of a straight line graph

REFERENCE IN OTHER RESOURCES

Head Start: Straight Line Graphs - Pages 26-30.

Alpha Workbooks: Straight Line Graphs - Pages 7-9.

The Equation of a Line

GRADIENT

The gradient of a line is a measure of the steepness of the line.

POSITIVE GRADIENT

increases from left to right

NEGATIVE GRADIENT

decreases from left to right

A basic formula for gradient is:

$$\text{GRADIENT} = \frac{\text{UP}}{\text{ACROSS}}$$

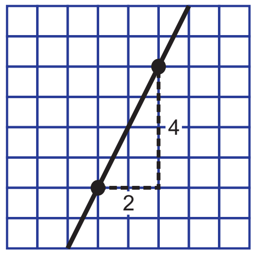
This is basic and you would have to be careful with signs.

Another way to get the gradient is to look at the line and find

- for each one moved across ⟶
- how many up ↑⊕ or down ↓⊖ has been moved? [?] this number is the gradient.

GRADIENTS FROM DIAGRAMS

EXAMPLE

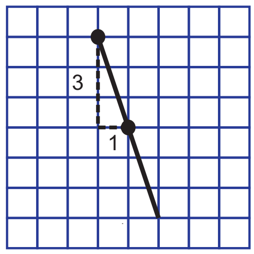


gradient $= \frac{4}{2} = 2$

or

for each one moved right the line moves up 2 squares

EXAMPLE



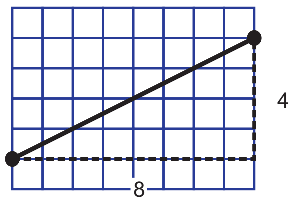
gradient $= -\frac{3}{1} = -3$

decreasing

or

for each one moved right the line moves down 3 squares

EXAMPLE



gradient $= \frac{4}{8} = \frac{1}{2}$

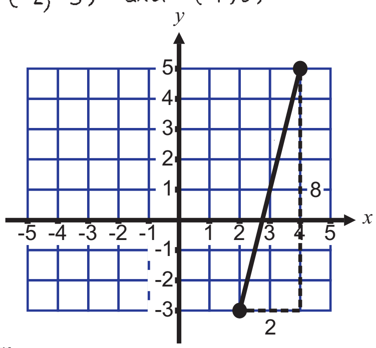
or

for each one moved right the line moves up $\frac{1}{2}$ a square

GRADIENTS FROM COORDINATES

EXAMPLE

Find the gradient of the line joining the points (2, −3) and (4, 5)



SOLUTION

From diagram gradient $= \frac{8}{2} = 4$

CALCULATING GRADIENTS FROM COORDINATES

To calculate a gradient from coordinates without a diagram use the formula:

$$\text{GRADIENT} = \frac{\text{CHANGE IN } y}{\text{CHANGE IN } x}$$

To avoid errors:

- write the coordinate with lower x first, how much does it increase by?
- what amount does y increase / decrease by?

CALCULATING GRADIENTS FROM COORDINATES

EXAMPLE

Find the gradient of the line joining the points (−1, 2) and (2, 17).

SOLUTION

Put (−1, 2) first – lower x value

(−1, 2) to (2, 17) change in x = +3

(−1, 2) to (2, 17) change in y = + 15

gradient $= \frac{+15}{+3} = 5$

EXAMPLE

Find the gradient of the line joining the points (2, 7) and (−3, 17).

SOLUTION

Put (−3, 17) first – lower x value

(−3, 17) to (2, 7) change in x = +5

(−3, 17) to (2, 7) change in y = −10

gradient $= \frac{-10}{+5} = -2$

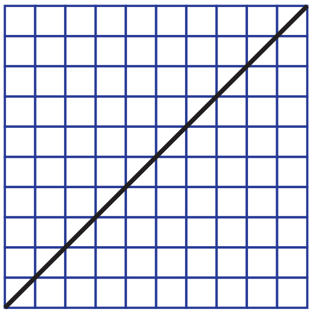
The Equation of a Line

Exercise A: Working out Gradients

The **gradient** of a line is a measure of the **steepness** of the line.

Find the **gradients** of the lines shown below:

(1)



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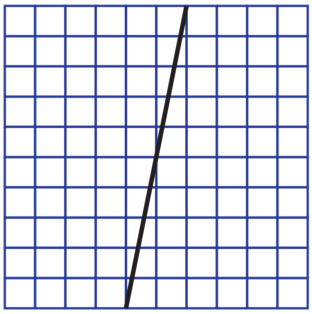

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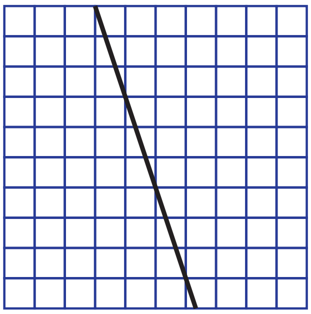

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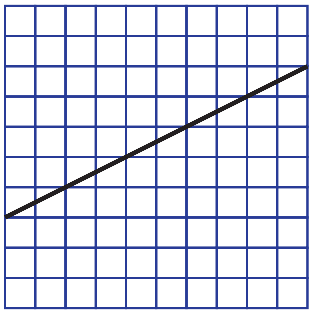

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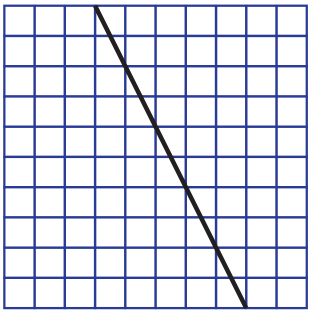

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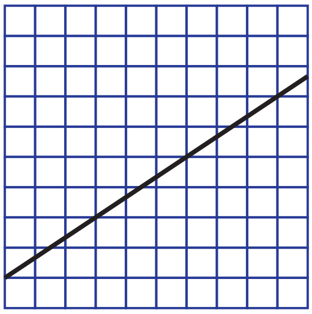

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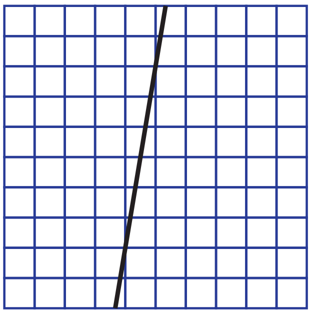

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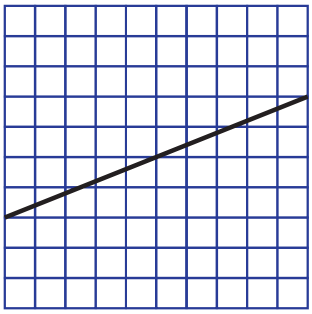


The Equation of a Line

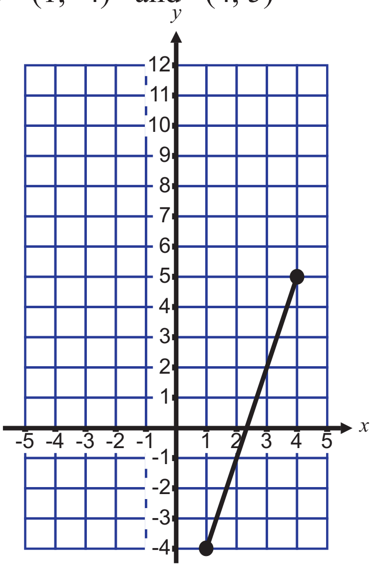
Exercise A: Working out Gradients

The **gradient** of a line is a measure of the **steepness** of the line.

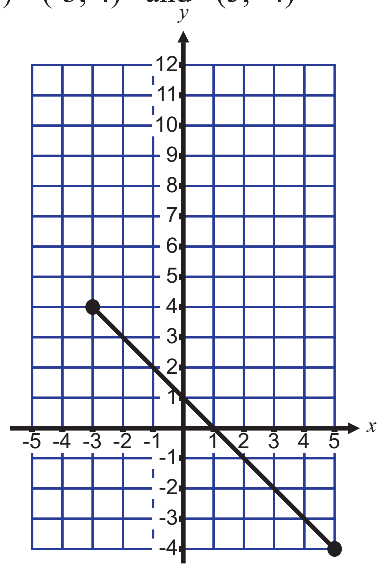
Find the **gradients** of the lines joining the points given below,

(use the diagram to check afterwards):

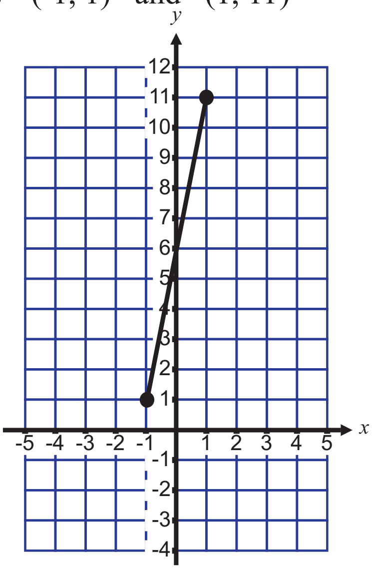
(9) (1, -4) and (4, 5)



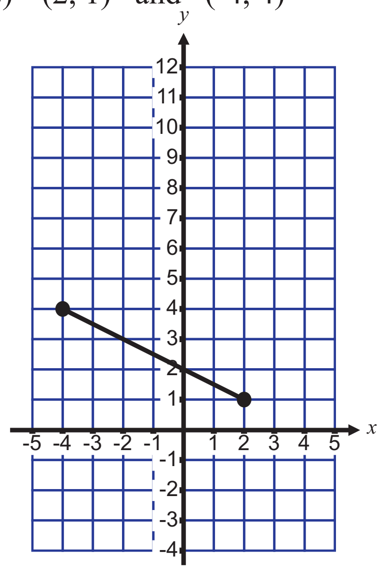
(10) (-3, 4) and (5, -4)



(11) (-1, 1) and (1, 11)



(12) (2, 1) and (-4, 4)



The **gradient** of a line is a measure of the **steepness** of the line.

Find the **gradients** of the lines joining the points given below,

(use the diagram to check afterwards):

(13) (3, -1) and (5, 9)

(14) (-4, 10) and (1, 0)

(15) (5, 12) and (3, 6)

(16) (-4, -3) and (2, 9)

The Equation of a Line

THE EQUATION OF A LINE (DIAGONAL)

The most useful form of the equation of a straight line graph is the $y = mx + c$ form.

This form directly tells you the gradient of the line and also where it crosses the y-axis (y-intercept).

$$y = mx + c$$

m, the number with the x is the GRADIENT!

c is the intercept with the y-axis (y-INTERCEPT).

PARALLEL LINES

Parallel lines have the same gradient!

GETTING INFORMATION FROM THE EQUATION

The gradient and y-intercept can be found providing the equation is in the form $y = mx + c$.

If the equation is not in this form, rearrange the equation to make y the subject.

EXAMPLE

Find the gradient and y-intercept of the following lines:

(a) $y = 2x + 1$ (b) $y = 3x - 5$ (c) $y = 4 - x$

(d) $y = -6x - 7$ (e) $2x + y = 4$ (f) $3x - 2y = 10$

SOLUTION

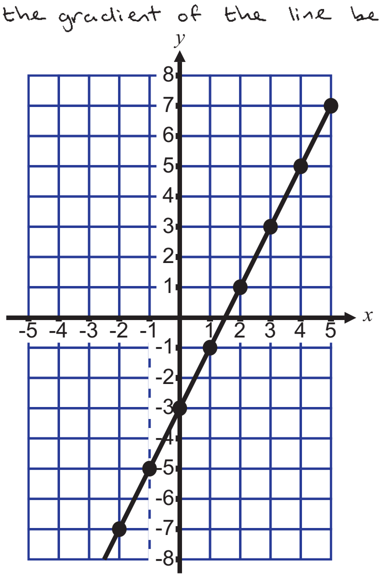
EQUATION	WORKING	GRADIENT	y-INTERCEPT
(a) $y = 2x + 1$	✓	2	1
(b) $y = 3x - 5$	✓	3	-5
(c) $y = 4 - x$	✓	-1	4
(d) $y = -6x - 7$	✓	-6	-7
(e) $2x + y = 4$	$y = 4 - 2x$	-2	4
(f) $3x - 2y = 10$	$3x = 10 + 2y$ $3x - 10 = 2y$ $\frac{3}{2}x - 5 = y$	$\frac{3}{2}$	-5

FINDING THE EQUATION OF A LINE

To find the equation of a line, obtain the gradient and the y-intercept.

EXAMPLE

Find the gradient of the line below:



SOLUTION

From graph:

gradient $= 2$

y-intercept $= -3$

$\rightarrow y = 2x - 3$

FINDING THE EQUATION OF A LINE

To find the equation of a line, obtain the gradient and y-intercept.

EXAMPLE

Find the equation of a line with a gradient of 3 and a y-intercept of 5, through $(0, 5)$.

SOLUTION

$y = 3x + 5$

EXAMPLE

Find the equation of a line which passes through the points $(-1, -6)$ and $(2, 9)$

SOLUTION

Find the gradient first:

$(-1, -6)$ to $(2, 9)$ change in $x = +3$

$(-1, -6)$ to $(2, 9)$ change in $y = +15$

gradient $= \frac{+15}{+3} = 5$

$\therefore y = 5x + c$ ①

Now put either coordinate into ①

Use $(2, 9)$ $9 = 5 \times 2 + c$

$c = -1$

$\therefore y = 5x - 1$

The Equation of a Line

<u>Exercise B:</u> The Equation of a Straight Line

The equation of a straight line graphs is $\boldsymbol{y = mx + c}$.

The value of $\boldsymbol{m}$ is the **gradient**.

The value of $\boldsymbol{c}$ is the ***y*-intercept**.

[Note: the form must have y as the subject *i.e.* $y = \ldots$]

(1) Complete the table to fill in the missing entries for:
the **equation**, **gradient** and ***y*-intercept**.

EQUATION	GRADIENT	*y*-INTERCEPT
$y = 3x + 7$		
	4	-2
$y = 10x - 5$		
$y = 4 - 2x$		
$y = 3x$		

(2) Complete the table to fill in the missing entries for:
the **equation**, **gradient** and ***y*-intercept**.

EQUATION	WORKING	GRADIENT	*y*-INTERCEPT
$x + y = 5$			
$x + 2y = 8$			
$3x - 2y = 4$			
$5x - 8y = 3$			
$6x + 2y - 15 = 0$			

The Equation of a Line

Exercise B: The Equation of a Straight Line

The equation of a straight line graphs is $\boldsymbol{y = mx + c}$.

The value of $\boldsymbol{m}$ is the **gradient**.

The value of $\boldsymbol{c}$ is the ***y*-intercept**.

[Note: the form must have y as the subject *i.e.* $y = \ldots$]

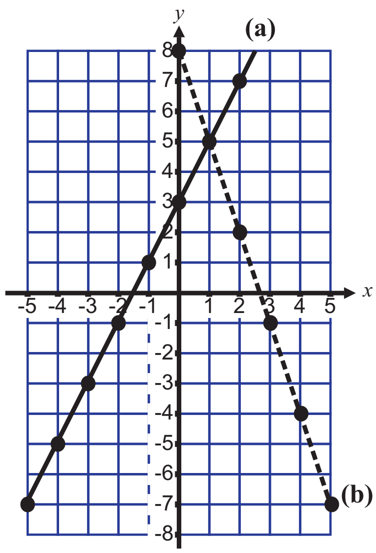
For the two **lines** in each question:

Determine their **gradients**,

State the ***y*-intercepts**,

Write the **equation** of each **line**.

(3)



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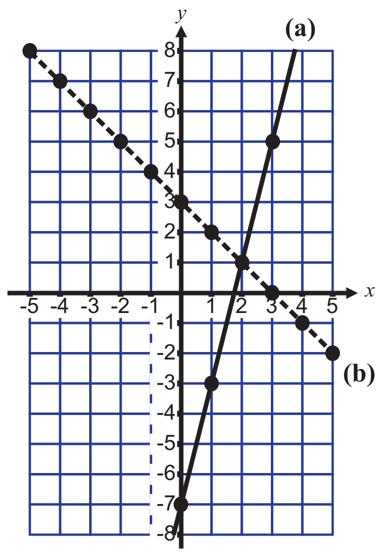

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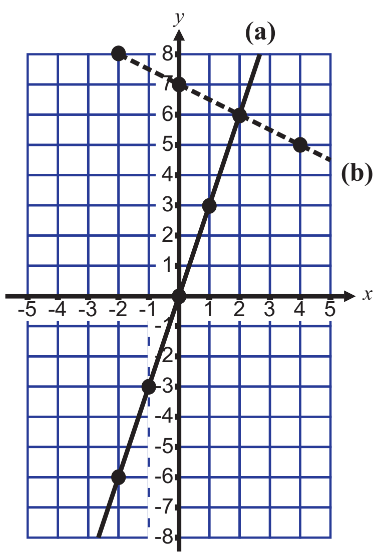

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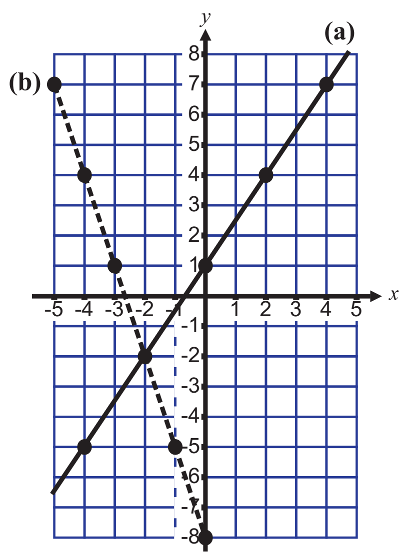


The Equation of a Line

<u>Exercise B:</u> The Equation of a Straight Line

The equation of a straight line graphs is $\boldsymbol{y = mx + c}$.

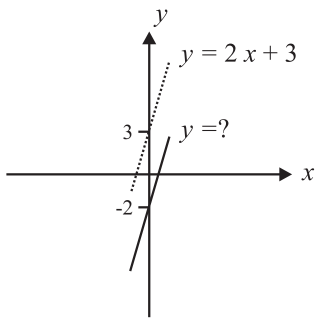
The value of $\boldsymbol{m}$ is the **gradient**.

The value of $\boldsymbol{c}$ is the $\boldsymbol{y}$**-intercept**.

[Note: the form must have y as the subject *i.e.* $y = \ldots$]

Find the **equation** of the **missing** line given the **equation** of the **parallel line**.

(7)



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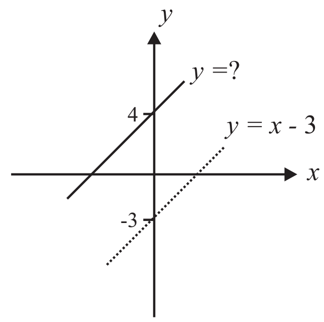

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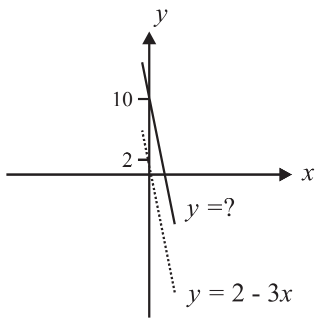

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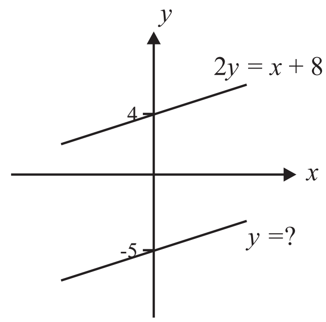


Find the **equation** of the **line** with the given **gradient** through the **given point**.

(11) gradient 2 passing through (0, 5)

(12) gradient 5 passing through (0, -3)

(13) gradient 1 passing through (0, 2)

(14) gradient -4 passing through (0, 7)

The Equation of a Line

<u>Exercise B:</u> The Equation of a Straight Line

The equation of a straight line graphs is $\boldsymbol{y = mx + c}$.

The value of $\boldsymbol{m}$ is the **gradient**.

The value of $\boldsymbol{c}$ is the ***y*-intercept**.

[Note: the form must have y as the subject *i.e.* $y = \ldots$]

Find the **equation** of the **line** with the given **gradient** through the **given point**.

(15) gradient 3 passing through (1, 7)

(16) gradient -2 passing through (5, 3)

Find the **equation** of the **line** that passes through the **given points**.

(17) (3, 4) and (5, 8)

(18) (-2, 1) and (3, 6)

(19) (-2, 12) and (1, 3)

(20) (2, 5) and (10, 9)

Parallel lines have the same **gradient**.

For each line in the question below, determine its gradient and find the pairs of lines that are parallel.

(21) $y = 3 - 2x$

$y = 3 - x$

$2x + y = 10$

$3x - 2y = 7$

$y = 1.5x + 6$

$x + y = 6$

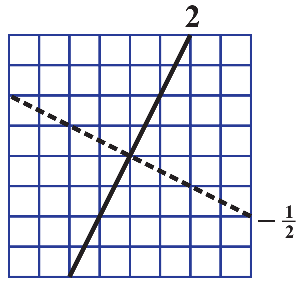
The Equation of a Line

<u>Exercise C:</u> Investigating Perpendicular Lines

Look at the **gradients** of these **lines** and the **perpendicular lines**.

Verify to yourself that the **gradients** are correct and write down the **relationship** between the **gradien**t of a **line** and is **perpendicular**.

(1)



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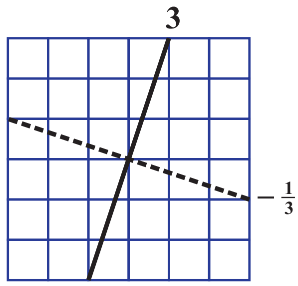

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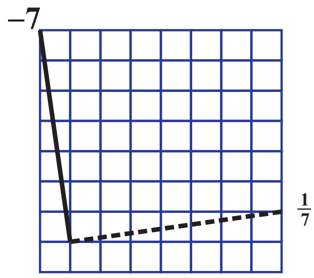

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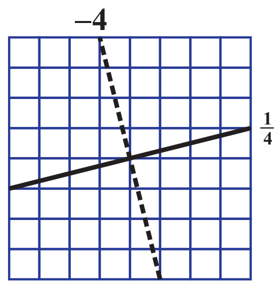

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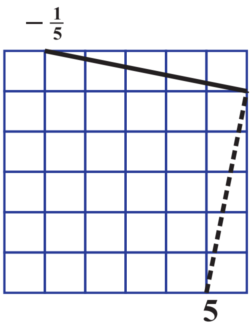

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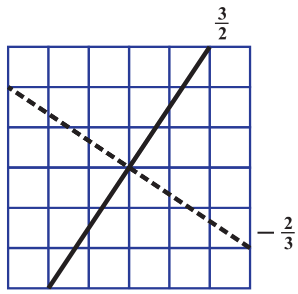

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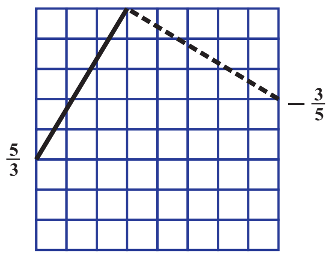

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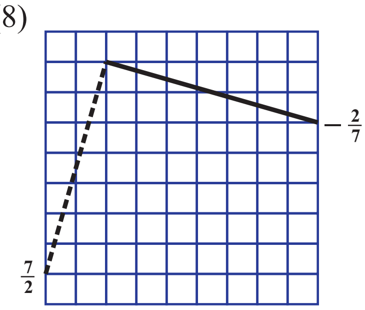

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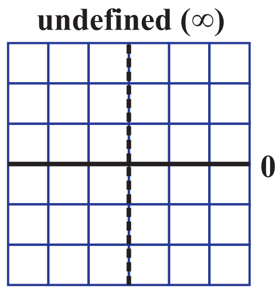


(10)

GRADIENT	PERPENDICULAR
2	$-\frac{1}{2}$
3	$-\frac{1}{3}$
-7	$\frac{1}{7}$
$\frac{1}{4}$	-4
$-\frac{1}{5}$	5
$\frac{3}{2}$	$-\frac{2}{3}$
$\frac{5}{3}$	$-\frac{3}{5}$
$-\frac{2}{7}$	$\frac{7}{2}$
0	undefined (∞)

RELATIONSHIP

To obtain the <u>perpendicular</u> to a <u>gradient</u> ......

The Equation of a Line

Exercise D: Perpendicular Lines

To find a **perpendicular** to a **gradient**:

- find the **reciprocal** of the **gradient**
- change the **sign**

The reciprocal of $\boldsymbol{b}$ is $\frac{1}{b}$; the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

Complete the table to give the **perpendicular gradients**:

	GRADIENT	PERPENDICULAR
(1)	5	
(2)	8	
(3)	-4	
(4)	$\frac{1}{3}$	
(5)	$-\frac{1}{2}$	
(6)	$\frac{5}{2}$	
(7)	$\frac{3}{10}$	
(8)	$-\frac{5}{4}$	

Identify whether these lines are **parallel**, **perpendicular** or **neither**.

(9) $y = -2x$
$y = \frac{1}{2}x + 1$

(10) $y = 5 - 3x$
$3x + y = 7$

(11) $y = 10 - 4x$
$4x - y = 12$

(12) $y = 4x + 1$
$y = \frac{1}{4}x - 3$

(13) $y = 2x + 1$
$y = -2x - 3$

(14) $3x + 2y = 4$
$2x - 3y = 4$

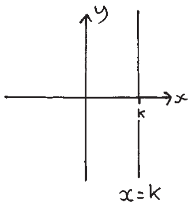
P. Leary

Lines Parallel to Axes

LINES PARALLEL TO AXES

Recall that lines parallel to axes do not contain both y and x !

VERTICAL LINE



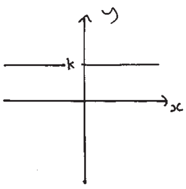
A vertical line has an equation of the form

$x = k$

The value of k is where the line cuts the x-axis.

Gradient: Infinite (∞)
UNDEFINED.

HORIZONTAL LINE



A horizontal line has an equation of the form

$y = k$

The value of k is where the line cuts the y-axis.

Gradient: zero! (0)

AXES

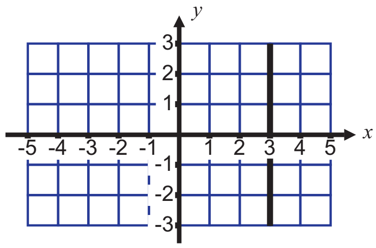
The x-axis has the equation $y=0$.

The y-axis has the equation $x=0$.

LINES PARALLEL TO AXES

EXAMPLE

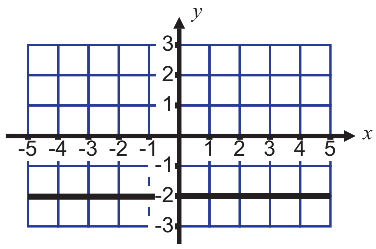
State the equation of this line:



SOLUTION $x = 3$

EXAMPLE

State the equation of this line:

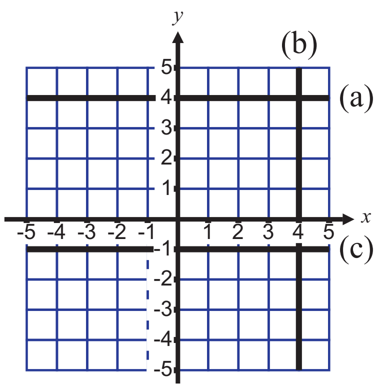


SOLUTION $y = -2$

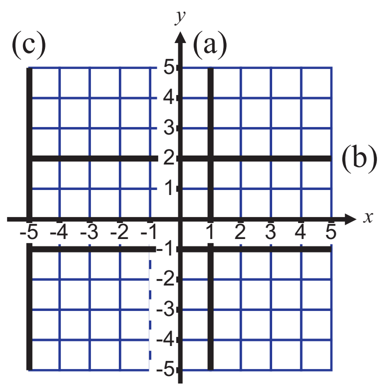
Exercise E: Lines Parallel to Axes

In each question write the equations for the three lines given:

(1)



(2)



THE EQUATION OF A LINE — EXERCISE A

(1) gradient $= 1$

(2) gradient $= 5$

(3) gradient $= -3$

(4) gradient $= \frac{1}{2}$

(5) gradient $= -2$

(6) gradient $= \frac{2}{3}$

(7) gradient $= 6$

(8) gradient $= \frac{2}{5}$

(9) change in $x \rightarrow +3$
change in $y \rightarrow +9$

gradient $= \frac{+9}{+3} = 3$

(10) change in $x \rightarrow +8$
change in $y \rightarrow -8$

gradient $= \frac{-8}{+8} = -1$

(11) change in $x \rightarrow +2$
change in $y \rightarrow +10$

gradient $= \frac{+10}{+2} = 5$

(12) change in $x \rightarrow +6$
change in $y \rightarrow -3$

gradient $= \frac{-3}{+6} = -\frac{1}{2}$

(13) gradient $= \frac{+10}{+2} = 5$

(14) gradient $= \frac{-10}{+5} = -2$

(15) gradient $= \frac{+6}{+2} = 3$

(16) gradient $= \frac{+12}{+6} = 2$

THE EQUATION OF A LINE — EXERCISE B

(1)

EQUATION	GRADIENT	y-INT.
$y = 3x+7$	3	7
$y = 4x-2$	4	-2
$y = 10x-5$	10	-5
$y = 4-2x$	-2	4
$y = 3x$	3	0

(2)

EQUATION	WORK	GRAD.	YINT
$x+y=5$	$y = 5-x$	-1	5
$x+2y=8$	$2y = 8-x$ $y = 4-\frac{1}{2}x$	$-\frac{1}{2}$	4
$3x-2y=4$	$3x = 4+2y$ $3x-4 = 2y$ $\frac{3}{2}x-2 = y$	$\frac{3}{2}$	-2
$5x-8y=3$	$5x = 3+8y$ $5x-3 = 8y$ $\frac{5}{8}x - \frac{3}{8} = y$	$\frac{5}{8}$	$-\frac{3}{8}$
$6x+2y-15=0$	$2y = 15-6x$ $y = \frac{15}{2} - 3x$	-3	$\frac{15}{2}$

(3) (a) $y = 2x+3$ (b) $y = -3x+8$

(4) (a) $y = 4x-7$ (b) $y = -x+3$

(5) (a) $y = 3x$ (b) $y = -\frac{1}{2}x+7$

(6) (a) $y = \frac{3}{2}x+1$ (b) $y = -3x-8$

THE EQUATION OF A LINE

EXERCISE B

(7) $y = 2x - 2$

(8) $y = x + 4$

(9) $y = 10 - 3x$

(10) $y = \frac{1}{2}x - 5$

(11) $y = 2x + 5$

(12) $y = 5x - 3$

(13) $y = x + 2$

(14) $y = -4x + 7$

(15) $y = 3x + c$

use $(1,7)$

$7 = 3 \times 1 + c$
$7 = 3 + c$
$4 = c$

$y = 3x + 4$

(16) $y = -2x + c$

use $(5,3)$

$3 = -2 \times 5 + c$
$3 = -10 + c$
$13 = c$

$y = -2x + 13$

(17) $m = \frac{4}{2} = 2$

$y = 2x + c$

use $(3,4)$

$4 = 2 \times 3 + c$
$4 = 6 + c$
$-2 = c$

$y = 2x - 2$

(18) $m = \frac{5}{5} = 1$

$y = x + c$

use $(3,6)$

$6 = 3 + c$
$3 = c$

$y = x + 3$

(19) $m = \frac{-9}{3} = -3$

$y = -3x + c$

use $(1,3)$

$3 = -3 \times 1 + c$
$3 = -3 + c$
$6 = c$

$y = -3x + 6$

(20) $m = \frac{4}{8} = \frac{1}{2}$

$y = \frac{1}{2}x + c$

use $(2,5)$

$5 = \frac{1}{2}(2) + c$
$5 = 1 + c$
$4 = c$

$y = \frac{1}{2}x + 4$

THE EQUATION OF A LINE — EXERCISE B

(21)

$y = 3 - 2x$	$\rightarrow$	$y = -2x + 3$	$m = -2$
$y = 3 - x$	$\rightarrow$	$y = -x + 3$	$m = -1$
$2x + y = 10$	$\rightarrow$	$y = -2x + 10$	$m = -2$
$3x - 2y = 7$	$\rightarrow$	$3x - 7 = 2y$ $\frac{3}{2}x - \frac{7}{2} = y$	$m = \frac{3}{2}$ (1.5)
$y = 1.5x + 6$	$\rightarrow$	$y = 1.5x + 6$	$m = 1.5$ $(\frac{3}{2})$
$x + y = 6$	$\rightarrow$	$y = -x + 6$	$m = -1$

Parallel pairs are $\boxed{y = 3 - 2x \text{ and } 2x + y = 10}$

$\boxed{y = 3 - x \text{ and } x + y = 6}$

$\boxed{3x - 2y = 7 \text{ and } y = 1.5x + 6}$

THE EQUATION OF A LINE — EXERCISE C

To obtain the perpendicular gradient....

- TAKE THE RECIPROCAL OF THE GRADIENT
- CHANGE THE SIGN

THE EQUATION OF A LINE EXERCISE D

(1) $-\frac{1}{5}$ (2) $-\frac{1}{8}$ (3) $\frac{1}{4}$ (4) -3

(5) 2 (6) $-\frac{2}{5}$ (7) $-\frac{10}{3}$ (8) $\frac{4}{5}$

(9) PERPENDICULAR
$m_1 = -2$
$m_2 = \frac{1}{2}$

(10) PARALLEL
$m_1 = -3$
$m_2 = -3$

(11) NEITHER
$m_1 = -4$
$m_2 = 4$

(12) NEITHER
$m_1 = 4$
$m_2 = \frac{1}{4}$

(13) NEITHER
$m_1 = 2$
$m_2 = -2$

(14) PERPENDICULAR
$m_1 = -3/2$
$m_2 = 2/3$

THE EQUATION OF A LINE EXERCISE E

(1) (a) $y=4$
(b) $x=4$
(c) $y=-1$

(2) (a) $x=1$
(b) $y=2$
(c) $x=-5$

SIMULTANEOUS EQUATIONS

Linear Simultaneous Equations

Objectives

★ know basic algebraic skills

- ☆ solve basic linear equations
- ☆ rearrange basic formulae
- ☆ expand brackets

★ recognise formulae that give straight line graphs

- ☆ $y = mx + c$
- ☆ $ax + by = k$

★ appreciate that when you solve linear simultaneous equations, you find where the lines cross

★ solve linear simultaneous equations algebraically by elimination

★ solve linear simultaneous equations algebraically by substitution

★ identify the best method to use (substitution or elimination)

REFERENCE IN OTHER RESOURCES

Head Start: Linear Simultaneous Equations - Pages 23-25.

Alpha Workbooks: Simultaneous Equations - Pages 17-19.

Simultaneous Equations - Linear Simultaneous Equations

LINEAR SIMULTANEOUS EQUATIONS

A linear formula is one which gives rise to a straight line graph.

Linear formulas contain both x and y (no power).

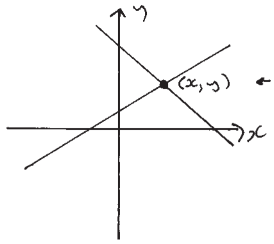
Algebraic manipulation can be used to manipulate between two key forms

- $y = mx + c$
- $ax + by = k$

Some equivalent formulas are presented in the table below:

$y = mx + c$ $\xleftrightarrow[\text{to}]{\text{rearranges}}$	$ax + by = k$
$y = x - 2$	$x - y = 2$
$y = -2x + 6$	$2x + y = 6$
$y = \frac{3}{2}x - 6$	$3x - 2y = 12$

When you solve linear simultaneous equations you are finding the crossing point of the straight line graphs.



← solving simultaneously gives this point!

LINEAR SIMULTANEOUS EQUATIONS

There are several ways of solving linear simultaneous equations.

GRAPH

Draw both graphs and look for the point of intersection.

This is a GCSE approach! An algebraic approach will be required at A-level.

These are:

ELIMINATION

This is the main method you will have met before.

Use if both equations are in the form $ax+by=k$ (x's and y's on same side).

You must make 'amount of y'* the same.

Then eliminate using
- same signs subtract
- different signs add

* or 'amount of x' the same.

SUBSTITUTION

Use if one or both equations are in the form $y = mx + c$.

Put the $y=$ equation into the other one.

NOTE

You can choose either algebraic method by rearranging formula.

EXAMPLE

Solve $2x + 3y = 6$ ①
$x - y = 8$ ②

by elimination.

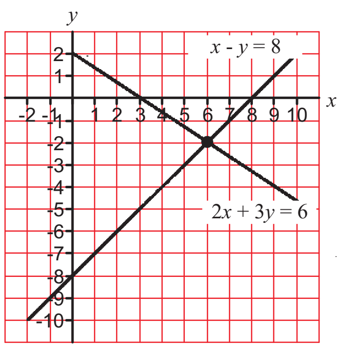
SOLUTION

$2x + 3y = 6$ ①
$x - y = 8$ ②

② × 3
$3x - 3y = 24$ ③
$2x + 3y = 6$ ① DIFFERENT SIGNS ADD

③ + ①
$5x = 30$
$x = 6$

in ①
$12 + 3y = 6$
$3y = -6$
$y = -2$



EXAMPLE

Solve $2x + 3y = 6$ ①
$y = x - 8$ ②

by substitution

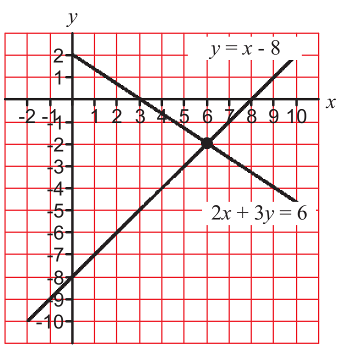
SOLUTION

$2x + 3y = 6$ ①
$y = x - 8$ ②

SUBSTITUTE ② in ①

$2x + 3(x - 8) = 6$
$2x + 3x - 24 = 6$
$5x - 24 = 6$
$5x = 30$
$x = 6$

in ② $y = 6 - 8 = -2$



EXAMPLE

Solve $3x + 2y = 10$ ①
$2x + y = 6$ ②

by elimination.

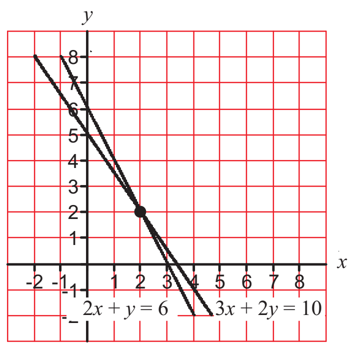
SOLUTION

$3x + 2y = 10$ ①
$2x + y = 6$ ②

② × 2
$4x + 2y = 12$ ③
$3x + 2y = 10$ ① SAME SIGNS SUBTRACT

③ − ①
$x = 2$

in ②
$4 + y = 6$
$y = 2$



EXAMPLE

Solve $3x + 2y = 10$ ①
$y = 6 - 2x$ ②

by substitution.

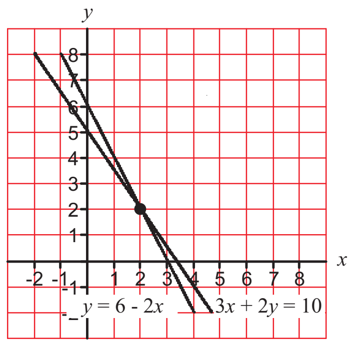
SOLUTION

$3x + 2y = 10$ ①
$y = 6 - 2x$ ②

SUBSTITUTE ② in ①

$3x + 2(6 - 2x) = 10$
$3x + 12 - 4x = 10$
$12 - x = 10$
$12 = 10 + x$
$2 = x$

in ② $y = 6 - 2 \times 2$
$y = 2$



Simultaneous Equations - Linear Simultaneous Equations

<u>Exercise:</u> Solving Linear Simultaneous Equations

When solving **linear simultaneous equations** you are finding the **point of intersection** of two **straight line graphs**.

There are two **algebraic methods** of **solving** them: **elimination** and **substitution**.

Try to recognise which method is easier, it depends on which form the equations are given in – the following questions break down the two methods.

Solve these **linear simultaneous equations** by the **elimination method**.

This is suitable as both equations are in the form $\boldsymbol{ax + by = k}$.

(1) $3x + y = 3$
$x + 2y = -4$

(2) $3x + 4y = 24$
$x + y = 7$

(3) $3x - 2y = 12$
$2x + y = 1$

(4) $2x - y = 4$
$3x - y = 5$

(5) $2x - 3y = 6$
$3x - 2y = 14$

(6) $5x + 2y = 54$
$2x - 5y = 10$

Solve these **linear simultaneous equations** by the **substitution method**.

This is suitable as at least one of the equations are in the form $\boldsymbol{y = mx + c}$.

(7) $2x + y = 10$
$y = x - 2$

(8) $4x + 5y = 5$
$y = 2x - 6$

(9) $3x + 2y = 12$
$y = x + 11$

(10) $y = 3x - 5$
$y = 2x - 2$

Either method can actually be used however the questions are presented – all you would need to do is **change the subject of the formula** to change between the methods (you can do this if you prefer one method but is may involve a lot more work and could make things quite difficult).

1) $3x + y = 3$ (1)
$x + 2y = -4$ (2)

(1) × 2
$6x + 2y = 6$ (3)
$x + 2y = -4$ (2)

SAME SIGNS SUBTRACT

(3) − (2)
$5x = 10$
$x = 2$

in (1)

$6 + y = 3$
$y = -3$

2) $3x + 4y = 24$ (1)
$x + y = 7$ (2)

(2) × 4
$4x + 4y = 28$ (3)
$3x + 4y = 24$ (1)

SAME SIGNS SUBTRACT

(3) − (1)
$x = 4$

in (2)

$4 + y = 7$
$y = 3$

3) $3x - 2y = 12$ (1)
$2x + y = 1$ (2)

(2) × 2
$4x + 2y = 2$ (3)
$3x - 2y = 12$ (1)

DIFFERENT SIGNS ADD

(3) + (1)
$7x = 14$
$x = 2$

in (1)

$4 + y = 1$
$y = -3$

4) $2x - y = 4$ (1)
$3x - y = 5$ (2)

SAME SIGNS SUBTRACT

(2) − (1)
$x = 1$

in (1)

$2 - y = 4$
$2 = y + 4$
$-2 = y$

REMEMBER

Before you eliminate make the 'amount of y' the same.

⑤ $2x - 3y = 6$ ①
$3x - 2y = 14$ ②

① ×2 $\quad 4x - 6y = 12$ ③
② ×3 $\quad 9x - 6y = 42$ ④

SAME SIGNS SUBTRACT

④ - ③
$$5x = 30$$
$$x = 6$$

in ①
$$12 - 3y = 6$$
$$3y = 6$$
$$y = 2$$

⑥ $5x + 2y = 54$ ①
$2x - 5y = 10$ ②

① ×5 $\quad 25x + 10y = 270$ ③
② ×2 $\quad 4x - 10y = 20$ ④

DIFFERENT SIGNS ADD

③ + ④
$$29x = 290$$
$$x = 10$$

in ①
$$50 + 2y = 54$$
$$2y = 4$$
$$y = 2$$

⑦ $2x + y = 10$ ①
$y = x - 2$ ②

SUBSTITUTE ② IN ①

$$2x + (x-2) = 10$$
$$3x - 2 = 10$$
$$3x = 12$$
$$x = 4$$

in ②
$$y = 4 - 2$$
$$y = 2$$

⑧ $4x + 5y = 5$ ①
$y = 2x - 6$ ②

SUBSTITUTE ② IN ①

$$4x + 5(2x-6) = 5$$
$$4x + 10x - 30 = 5$$
$$14x - 30 = 5$$
$$14x = 35$$
$$x = \frac{35}{14} = 2.5$$

in ②
$$y = 2 \times 2.5 - 6$$
$$y = 5 - 6$$
$$y = -1$$

(9) $3x + 2y = 12$ (1)
$y = x + 11$ (2)

SUBSTITUTE (2) IN (1)

$$3x + 2(x + 11) = 12$$

$$3x + 2x + 22 = 12$$

$$5x + 22 = 12$$

$$5x = -10$$

$$x = -2$$

in (1)

$y = -2 + 11$
$y = 9$

(10) $y = 3x - 5$ (1)
$y = 2x - 2$ (2)

SUBSTITUTE FOR y (EQUATE)

$$3x - 5 = 2x - 2$$

$$3x - 2x - 5 = -2$$

$$x - 5 = -2$$

$$x = 3$$

in (1)

$y = 3 \times 3 - 5$
$y = 4$

SURDS

Surds

Objectives

★ know key language associated with number: integer, rational and irrational

★ recognise surds

★ know the surd laws: multiplication and division only

★ simplify surds

★ rationalise the denominator is fractions containing surds

★ use brackets and algebraic techniques in solving mixed problems with surds

REFERENCE IN OTHER RESOURCES

Head Start: Types of Number - Pages 1-2 (Surd Rules Not Covered).

Alpha Workbooks: Types of Number - Page 3 (Surd Rules Not Covered).

Surds

TYPES OF NUMBER

INTEGERS – are whole numbers

RATIONAL NUMBERS – are numbers that can be expressed exactly as a fraction
e.g. $0.7 = \frac{7}{10}$, $0.25 = \frac{1}{4}$, $0.17 = \frac{17}{100}$

$0.\dot{6} = \frac{2}{3}$, $0.\dot{1}0\dot{5} = \frac{35}{333}$

IRRATIONAL NUMBERS – are numbers that cannot be expressed exactly as a fraction
e.g. $\sqrt{2}$, $\sqrt{11}$, $\sqrt[3]{5}$, π

IRRATIONAL NUMBERS include:

- square roots of numbers that are not square numbers
- cube roots of numbers that are not cube numbers

ROOTS that give rise to IRRATIONAL NUMBERS are called SURDS.

SURD RULES

The surd rules allow you to re-write surds which are multiplied or divided.

MULTIPLICATION RULE	DIVISION RULE
$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$	$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

The same ideas do not work for addition and subtraction.

BASIC SIMPLIFYING AND USE OF LAWS

EXAMPLE

Simplify $\sqrt{3} \times \sqrt{3}$

SOLUTION

$\sqrt{3} \times \sqrt{3} = \sqrt{3 \times 3} = \sqrt{9} = 3$

you should be able to come up with this answer directly without the rule (squaring removes the root).

EXAMPLE

Simplify $2\sqrt{3} \times 5\sqrt{3}$

SOLUTION

$2\sqrt{3} \times 5\sqrt{3}$ reorder $= (2 \times 5) \times (\sqrt{3} \times \sqrt{3}) = 10 \times 3 = 30$

EXAMPLE

Calculate $(2\sqrt{7})^2$

SOLUTION

$(2\sqrt{7})^2 = 2\sqrt{7} \times 2\sqrt{7} = (2 \times 2) \times (\sqrt{7} \times \sqrt{7}) = 4 \times 7 = 28$

EXAMPLE

Evaluate $\sqrt{2} \times \sqrt{50}$

SOLUTION

$\sqrt{2} \times \sqrt{50} = \sqrt{2 \times 50} = \sqrt{100} = 10$

EXAMPLE

Write as a single surd $\sqrt{5} \times \sqrt{7}$

SOLUTION

$\sqrt{5} \times \sqrt{7} = \sqrt{5 \times 7} = \sqrt{35}$

SIMPLIFYING SURDS

When simplifying a single surd:

- square roots – look for factors that are square numbers
- cube roots – look for factors that are cube numbers

EXAMPLE

Simplify $\sqrt{12}$

SOLUTION

$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

EXAMPLE

Simplify $\sqrt{45}$

SOLUTION

$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$

When simplifying two surds which are added/subtracted, simplify each surd first then collect like terms (surds).

EXAMPLE

Simplify $\sqrt{12} + \sqrt{75}$

SOLUTION

$\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

$\sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

$\therefore \sqrt{12} + \sqrt{75} = 2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

RATIONALISING THE DENOMINATOR

Rationalising the denominator means re-writing a fraction so the bottom number does not contain a surd.

Use laws of equivalent fractions to do this.

EXAMPLE

Rationalise the denominator $\frac{1}{\sqrt{2}}$

FOR SINGLE TERM JUST MULTIPLY BY SURD ON BOTTOM

SOLUTION

$\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$

EXAMPLE

Rationalise the denominator $\frac{2\sqrt{3}}{\sqrt{5}}$

SOLUTION

$\frac{2\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{3} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{2\sqrt{3}\sqrt{5}}{5} = \frac{2\sqrt{15}}{5}$

EXAMPLE

Rationalise the denominator $\frac{\sqrt{2}+1}{\sqrt{2}}$

SOLUTION

$\frac{\sqrt{2}+1}{\sqrt{2}} = \frac{\sqrt{2}+1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}(\sqrt{2}+1)}{2} = \frac{\sqrt{2}\sqrt{2}+1\sqrt{2}}{2}$

$= \frac{2+\sqrt{2}}{2}$

Surds

Exercise A: Basic Surds and Use of Laws in Simplifying

A **rational number** is a number that can be written as a **fraction**.

An **irrational number** is one which **cannot** be expressed exactly as a **fraction**.

An expression involving an **irrational number** is called a **surd**.

The **surd rules** allow you to re-write **surds** which are **multiplied** or **divided**.

The Multiplication Rule	***The Division Rule***
$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$	$\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

The same ideas **<u>do not</u>** work for **addition** and **subtraction**.

Exercise 1

Work these out; the answer will be an **integer**!

(1) $\sqrt{2} \times \sqrt{2}$ (2) $\sqrt{5} \times \sqrt{5}$ (3) $\sqrt{11} \times \sqrt{11}$ (4) $\left(\sqrt{8}\right)^2$

(5) $2\sqrt{3} \times \sqrt{3}$ (6) $\sqrt{6} \times 4\sqrt{6}$ (7) $3\sqrt{7} \times 2\sqrt{7}$ (8) $\left(2\sqrt{5}\right)^2$

Use the **multiplication law** to write these as a **single surd** and **simplify** to a **rational number**, in most cases the result will be an **integer**.

(9) $\sqrt{5} \times \sqrt{20}$ (10) $\sqrt{2} \times \sqrt{8}$ (11) $\sqrt{27} \times \sqrt{3}$ (12) $\sqrt{50} \times \sqrt{2}$

(13) $\sqrt{12} \times \sqrt{3}$ (14) $\sqrt{2} \times \sqrt{32}$ (15) $\dfrac{\sqrt{75}}{\sqrt{3}}$ (16) $\dfrac{\sqrt{72}}{\sqrt{2}}$

Use the **multiplication law** to write these as a **single surd** (the result is **irrational**).

(17) $\sqrt{3} \times \sqrt{5}$ (18) $\sqrt{7} \times \sqrt{2}$ (19) $\sqrt{6} \times \sqrt{3}$ (20) $\sqrt{5} \times \sqrt{6}$

Surds

Exercise B: Simplifying Surds

When **simplifying** a **single surd** you need to make the number inside the **root** as **small** as possible.

For **square roots** look for **factors** that are **square numbers** and break the **surd** apart and then **simplify**.

When you are asked to **simplify two surds** that are added or subtracted, **simplify** each **surd** first and you should obtain **like terms** (**surds**) which can be **collected**.

Exercise 2

Simplify these **surds** (the result is **irrational**).

(1) $\sqrt{8}$	(2) $\sqrt{75}$	(3) $\sqrt{18}$	(4) $\sqrt{50}$
(5) $\sqrt{28}$	(6) $\sqrt{490}$	(7) $\sqrt{125}$	(8) $\sqrt{63}$
(9) $\sqrt{40}$	(10) $\sqrt{600}$	(11) $\sqrt{98}$	(12) $\sqrt{405}$
(13) $\sqrt{32}$	(14) $\sqrt{80}$	(15) $\sqrt{72}$	(16) $\sqrt{162}$

Simplify these **surds** (the result is **irrational**).

(17) $\sqrt{2}+\sqrt{8}$	(18) $\sqrt{75}-\sqrt{12}$	(19) $\sqrt{24}+\sqrt{600}$	(20) $\sqrt{45}+\sqrt{180}$

Exercise C: Rationalising the Denominator

Some times **surds** appear in **fractions**.

Rationalising the **denominator** means **re-writing** the **fraction** so the **bottom number** does not contain a **surd** (*i.e.* the **denominator** is **rational**).

Exercise 3

Rationalise the denominator in these **surds**, **simplifying** the answer.

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{4}{\sqrt{5}}$ (3) $\frac{7}{\sqrt{3}}$ (4) $\frac{10}{\sqrt{7}}$

(5) $\frac{11}{\sqrt{15}}$ (6) $\frac{6}{\sqrt{3}}$ (7) $\frac{12}{\sqrt{6}}$ (8) $\frac{1}{\sqrt{2}}$

(9) $\frac{2\sqrt{2}}{\sqrt{3}}$ (10) $\frac{5\sqrt{7}}{\sqrt{2}}$ (11) $\frac{\sqrt{5}}{\sqrt{11}}$ (12) $\frac{2\sqrt{15}}{\sqrt{5}}$

Rationalise the denominator in these **surds**, **simplifying** the answer.

(13) $\frac{2}{3\sqrt{8}}$ (14) $\frac{6\sqrt{10}}{\sqrt{75}}$ (15) $\frac{12+\sqrt{2}}{\sqrt{2}}$ (16) $\frac{\sqrt{3}-2}{\sqrt{5}}$

Exercise D: Mixed Problems Involving Surds

Evaluate these squares

(1) $(\sqrt{3})^2$

(2) $(2\sqrt{5})^2$

(3) $(6\sqrt{2})^2$

(4) $(4\sqrt{10})^2$

Multiply out the brackets and give the result in the form $a + b\sqrt{c}$

(5) $(2+\sqrt{3})(3+\sqrt{3})$

(6) $(7+\sqrt{2})(4-\sqrt{2})$

(7) $(6-\sqrt{5})(4+2\sqrt{5})$

(8) $(2-2\sqrt{2})(3-3\sqrt{2})$

Multiply out the brackets and give the result in the form $a + b\sqrt{c}$

(9) $(4+\sqrt{2})(4-\sqrt{2})$

(10) $(8+\sqrt{5})(8-\sqrt{5})$

(11) Given that $a = 3 + 2\sqrt{6}$ and $b = 2 - \sqrt{6}$, evaluate

(a) $a + b$

(b) $a - b$

(c) a^2

(d) ab

(12) Given that $x = 5 + \sqrt{7}$ and $y = 5 - \sqrt{7}$, evaluate

(a) $x + y$

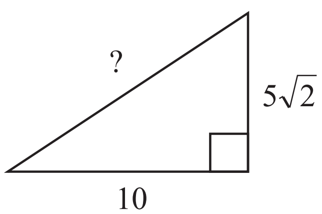
(b) $x - y$

(c) x^2

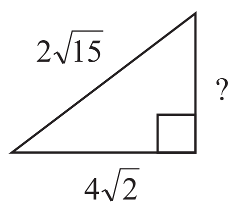
(d) xy

Work out the missing side in each triangle

(13)



(14)



SURDS — EXERCISE 1

① $\sqrt{2}\times\sqrt{2} = 2$

② $\sqrt{5}\times\sqrt{5} = 5$

③ $\sqrt{11}\times\sqrt{11} = 11$

④ $(\sqrt{8})^2 = \sqrt{8}\times\sqrt{8} = 8$

⑤ $2\sqrt{3}\times\sqrt{3} = 2\times 3 = 6$

⑥ $\sqrt{6}\times 4\sqrt{6} = 4\times 6 = 24$

⑦ $3\sqrt{7}\times 2\sqrt{7} = 6\times 7 = 42$

⑧ $(2\sqrt{5})^2 = 2\sqrt{5}\times 2\sqrt{5} = 4\times 5 = 20$

⑨ $\sqrt{5}\times\sqrt{20} = \sqrt{100} = 10$

⑩ $\sqrt{2}\times\sqrt{8} = \sqrt{16} = 4$

⑪ $\sqrt{27}\times\sqrt{3} = \sqrt{81} = 9$

⑫ $\sqrt{50}\times\sqrt{2} = \sqrt{100} = 10$

⑬ $\sqrt{12}\times\sqrt{3} = \sqrt{36} = 6$

⑭ $\sqrt{2}\times\sqrt{32} = \sqrt{64} = 8$

⑮ $\sqrt{\frac{75}{3}} = \sqrt{25} = 5$

⑯ $\frac{\sqrt{72}}{\sqrt{2}} = \sqrt{\frac{72}{2}} = \sqrt{36} = 6$

⑰ $\sqrt{3}\times\sqrt{5} = \sqrt{15}$

⑱ $\sqrt{7}\times\sqrt{2} = \sqrt{14}$

⑲ $\sqrt{6}\times\sqrt{3} = \sqrt{18}$

⑳ $\sqrt{5}\times\sqrt{6} = \sqrt{30}$

SURDS — EXERCISE 2

① $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

② $\sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

③ $\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

④ $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$

⑤ $\sqrt{28} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

⑥ $\sqrt{490} = \sqrt{49}\sqrt{10} = 7\sqrt{10}$

⑦ $\sqrt{125} = \sqrt{25}\sqrt{5} = 5\sqrt{5}$

⑧ $\sqrt{63} = \sqrt{9}\sqrt{7} = 3\sqrt{7}$

⑨ $\sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$

⑩ $\sqrt{600} = \sqrt{100}\sqrt{6} = 10\sqrt{6}$

⑪ $\sqrt{98} = \sqrt{49}\sqrt{2} = 7\sqrt{2}$

⑫ $\sqrt{405} = \sqrt{81}\sqrt{5} = 9\sqrt{5}$

⑬ $\sqrt{32} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$

⑭ $\sqrt{80} = \sqrt{16}\sqrt{5} = 4\sqrt{5}$

⑮ $\sqrt{72} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$

⑯ $\sqrt{162} = \sqrt{81}\sqrt{2} = 9\sqrt{2}$

⑰ $\sqrt{2}+\sqrt{8} = \sqrt{2}+2\sqrt{2} = 3\sqrt{2}$

⑱ $\sqrt{75}-\sqrt{12} = 5\sqrt{3}-2\sqrt{3} = 3\sqrt{3}$

⑲ $\sqrt{24}+\sqrt{600} = 2\sqrt{6}+10\sqrt{6} = 12\sqrt{6}$

⑳ $\sqrt{45}+\sqrt{180} = 3\sqrt{5}+6\sqrt{5} = 9\sqrt{5}$

SURDS — EXERCISE 3

① $\frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{3\sqrt{2}}{2}$

② $\frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{4\sqrt{5}}{5}$

③ $\frac{7}{\sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{7\sqrt{3}}{3}$

④ $\frac{10}{\sqrt{7}} = \frac{10\sqrt{7}}{\sqrt{7}\sqrt{7}} = \frac{10\sqrt{7}}{7}$

⑤ $\frac{11}{\sqrt{15}} = \frac{11\sqrt{15}}{\sqrt{15}\sqrt{15}} = \frac{11\sqrt{15}}{15}$

⑥ $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

⑦ $\frac{12}{\sqrt{6}} = \frac{12\sqrt{6}}{\sqrt{6}\sqrt{6}} = \frac{12\sqrt{6}}{6} = 2\sqrt{6}$

⑧ $\frac{1}{\sqrt{2}} = \frac{1\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$

⑨ $\frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{6}}{3}$

⑩ $\frac{5\sqrt{7}}{\sqrt{2}} = \frac{5\sqrt{7}\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{5\sqrt{14}}{2}$

⑪ $\frac{\sqrt{5}}{\sqrt{11}} = \frac{\sqrt{5}\sqrt{11}}{\sqrt{11}\sqrt{11}} = \frac{\sqrt{55}}{11}$

⑫ $\frac{2\sqrt{15}}{\sqrt{5}} = \frac{2\sqrt{15}\sqrt{5}}{5} = \frac{2\sqrt{75}}{5} =$

$\frac{2\sqrt{25}\sqrt{3}}{5} = \frac{10\sqrt{3}}{5} = 2\sqrt{3}$

or!!! $2\sqrt{\frac{15}{5}} = 2\sqrt{3}$!!

⑬ $\frac{2}{3\sqrt{8}} = \frac{2}{3\times 2\sqrt{2}} = \frac{2}{6\sqrt{2}} = \frac{1}{3\sqrt{2}} = \frac{1\sqrt{2}}{3\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{3\times 2} = \frac{\sqrt{2}}{6}$

SIMPLIFY FIRST — RATIONALISE

⑭ $\frac{6\sqrt{10}}{\sqrt{75}} = \frac{6\sqrt{10}}{\sqrt{25}\sqrt{3}} = \frac{6\sqrt{10}}{5\sqrt{3}} = \frac{6\sqrt{10}\sqrt{3}}{5\sqrt{3}\sqrt{3}} = \frac{6\sqrt{30}}{15} = \frac{2\sqrt{30}}{5}$

SIMPLIFY FIRST — RATIONALISE

⑮ $\frac{12+\sqrt{2}}{\sqrt{2}} = \frac{(12+\sqrt{2})\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{12\sqrt{2}+2}{2} = 6\sqrt{2}+1$

⑯ $\frac{\sqrt{3}-2}{\sqrt{5}} = \frac{(\sqrt{3}-2)\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{\sqrt{15}-2\sqrt{5}}{5}$

SURDS

EXERCISE 4

① $(\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = 3$

② $(2\sqrt{5})^2 = 2\sqrt{5} \times 2\sqrt{5} = 2 \times 2 \times \sqrt{5} \times \sqrt{5}$
$= 4 \times 5 \quad = 20$

③ $(6\sqrt{2})^2 = 6\sqrt{2} \times 6\sqrt{2} = 6 \times 6 \times \sqrt{2} \times \sqrt{2}$
$= 36 \times 2 \quad = 72$

④ $(4\sqrt{10})^2 = 4\sqrt{10} \times 4\sqrt{10} = 4 \times 4 \times \sqrt{10} \times \sqrt{10}$
$= 16 \times 10 \quad = 160$

⑤ $(2+\sqrt{3})(2+\sqrt{3})$
$= 4 + 2\sqrt{3} + 2\sqrt{3} + 3$
$= 7 + 4\sqrt{3}$

⑥ $(7+\sqrt{2})(4-\sqrt{2})$
$= 28 - 7\sqrt{2} + 4\sqrt{2} - 2$
$= 26 - 3\sqrt{2}$

⑦ $(6-\sqrt{5})(4+2\sqrt{5})$
$= 24 + 12\sqrt{5} - 4\sqrt{5} - 10$
$= 14 + 8\sqrt{5}$

⑧ $(2-2\sqrt{2})(3-3\sqrt{2})$
$= 6 - 6\sqrt{2} - 6\sqrt{2} + 12$
$= 18 - 12\sqrt{2}$

⑨ $(4+\sqrt{2})(4-\sqrt{2})$
$= 16 - 4\sqrt{2} + 4\sqrt{2} - 2$
$= 14$

⑩ $(8+\sqrt{5})(8-\sqrt{5})$
$= 64 - 8\sqrt{5} + 8\sqrt{5} - 5$
$= 59$

⑪ (a) $a + b = (3+2\sqrt{6}) + (2-\sqrt{6}) = 5 + \sqrt{6}$

(b) $a - b = (3+2\sqrt{6}) - (2-\sqrt{6}) = 1 + 3\sqrt{6}$

(c) $a^2 = (3+2\sqrt{6})^2 = (3+2\sqrt{6})(3+2\sqrt{6}) = 9 + 6\sqrt{6} + 6\sqrt{6} + 24 = 33 + 12\sqrt{6}$

(d) $ab = (3+2\sqrt{6})(2-\sqrt{6}) = 6 - 3\sqrt{6} + 4\sqrt{6} - 12 = -6 + \sqrt{6}$

⑫ (a) $x + y = (5+\sqrt{7}) + (5-\sqrt{7}) = 10$

(b) $x - y = (5+\sqrt{7}) - (5-\sqrt{7}) = 2\sqrt{7}$

(c) $x^2 = (5+\sqrt{7})^2 = (5+\sqrt{7})(5+\sqrt{7}) = 25 + 5\sqrt{7} + 5\sqrt{7} + 7 = 32 + 10\sqrt{7}$

(d) $xy = (5+\sqrt{7})(5-\sqrt{7}) = 25 - 5\sqrt{7} + 5\sqrt{7} - 7 = 18$

⑬ $?^2 = 10^2 + (5\sqrt{2})^2$
$?^2 = 100 + (25 \times 2)$
$?^2 = 150$
$? = \sqrt{150}$
$? = \sqrt{25}\sqrt{6}$
$? = 5\sqrt{6}$

⑭ $?^2 = (2\sqrt{15})^2 - (4\sqrt{2})^2$
$?^2 = (4 \times 15) - (16 \times 2)$
$?^2 = 60 - 32$
$?^2 = 28$
$? = \sqrt{28}$
$? = \sqrt{4}\sqrt{7}$
$? = 2\sqrt{7}$

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QUADRATICS

Factorising Quadratics and Solving Equations

Objectives

★ multiply out brackets

★ factorise quadratic expressions

★ recognise quadratic forms which will factorise quickly, special cases:

- ☆ quadratics with x as a common factor (two term)
- ☆ the difference between two squares (two term)
- ☆ quadratics with a common numerical factor throughout

★ appreciate that not all quadratics will factorise

★ solve quadratic equations by factorising

★ know that the solutions to a quadratic equation are the roots of the quadratic graph

★ be aware of different ways of solving quadratic equations

REFERENCE IN OTHER RESOURCES

Head Start: Factorising Quadratics and Equations - Pages 17-22.

Alpha Workbooks: Factorising Quadratics and Equations - Pages 20-23.

Factorising Quadratics and Solving Equations

QUADRATICS

A quadratic is a polynomial expression with the highest power of two (order 2).

These are examples of quadratics:

$$x^2 - 25$$
$$x^2 + 5x + 6$$
$$3x^2 + 12x$$
$$2x^2 + 7x + 3$$
$$9 - x^2$$
$$2x^2 + 10x + 12$$

FACTORISING QUADRATICS

Factorising quadratics means re-writing the quadratic expression using brackets.

QUADRATIC	FACTORISED QUADRATIC
$x^2 - 25$	$(x+5)(x-5)$
$x^2 + 5x + 6$	$(x+3)(x+2)$
$3x^2 + 12x$	$3x(x+4)$
$2x^2 + 7x + 3$	$(2x+1)(x+3)$
$9 - x^2$	$(3+x)(3-x)$
$2x^2 + 10x + 12$	$2(x+3)(x+2)$

In this section, it will be assumed that you can expand brackets (using methods like FOIL).

REASONS FOR FACTORISING QUADRATICS

A vital skill in AS/A2 Maths is the ability to factorise a quadratic.

Factorising a quadratic is used in:

- solving quadratic equations
- sketching graphs of quadratic functions

COMMENT

There are other ways to do these problems but factorising is often easiest when you can confidently factorise.

However,

IT IS IMPORTANT THAT YOU REALISE THAT NOT ALL QUADRATICS CAN BE FACTORISED.

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Factorising Quadratics and Solving Equations

BASIC I : TWO TERM ! x AS A COMMON FACTOR

① $x^2 + 5x$
$x(x+5)$

② $x^2 - 7x$
$x(x-7)$

③ $x^2 - 10x$
$x(x-10)$

④ $2x^2 + 8x$
$2x(x+4)$

⑤ $3x^2 - 9x$
$3x(x-3)$

⑥ $10x^2 + 35x$
$5x(2x+7)$

$x^2 + 5x$ TWO TERM!
x AS A COMMON FACTOR
TAKE x OUT
$x(x+5)$

$x^2 - 7x$ TWO TERM!
x AS A COMMON FACTOR
TAKE x OUT
$x(x-7)$

$x^2 - 10x$ TWO TERM!
x AS A COMMON FACTOR
TAKE x OUT
$x(x-10)$

$2x^2 + 8x$ TWO TERM!
x AS A COMMON FACTOR
2 COMMON NUMERICAL FACTOR
TAKE $2x$ OUT
$2x(x+4)$

> SPECIAL CASE !
> x AS A COMMON FACTOR
> LOOK FOR • QUADRATIC WITH TWO TERMS
> • BOTH TERMS CONTAIN x

BASIC II : TWO TERM ! DIFFERENCE BETWEEN TWO SQUARES

① $x^2 - 4$
$(x+2)(x-2)$

② $x^2 - 25$
$(x+5)(x-5)$

③ $x^2 - 49$
$(x+7)(x-7)$

④ $9 - x^2$
$(3+x)(3-x)$

⑤ $4x^2 - 81$
$(2x+9)(2x-9)$

⑥ $16x^2 - 121y^2$
$(4x+11y)(4x-11y)$

$x^2 - 4$ TWO TERM!
DIFFERENCE BETWEEN TWO SQUARES
$(x+2)(x-2)$

$x^2 - 25$ TWO TERM!
DIFFERENCE BETWEEN TWO SQUARES
$(x+5)(x-5)$

$4x^2 - 81$ TWO TERM!
DIFFERENCE BETWEEN TWO SQUARES
$(2x+9)(2x-9)$

$16x^2 - 121y^2$ TWO TERM!
DIFFERENCE BETWEEN TWO SQUARES
$(4x+11y)(4x-11y)$

> SPECIAL CASE !
> DIFFERENCE BETWEEN TWO SQUARES
> LOOK FOR • QUADRATIC WITH TWO TERMS
> • DIFFERENCE ⊖
> • SQUARE TERMS
>
> AND KNOW KEY RESULT
> $x^2 - a^2 = (x+a)(x-a)$

Factorising Quadratics and Solving Equations

GENERAL I: THREE TERM! (COEFFICIENT OF x^2 IS ONE)

① $x^2+8x+15$
$(x+3)(x+5)$

② $x^2+9x+14$
$(x+2)(x+7)$

③ $x^2+14x+49$
$(x+7)(x+7)$

④ $x^2-7x+10$
$(x-2)(x-5)$

⑤ $x^2-7x+12$
$(x-3)(x-4)$

⑥ $x^2-15x+54$
$(x-6)(x-9)$

⑦ x^2+2x-3
$(x+3)(x-1)$

⑧ $x^2+4x-21$
$(x+7)(x-3)$

⑨ $x^2-3x-10$
$(x+2)(x-5)$

$x^2 \quad + 8x \quad + 15$

BOTH ⊕ SAME SIGNS

$x^2 \quad + 8x \quad + 15$

x AND x +1 AND +15; +3 AND +5

$(x+1)(x+15) = x^2+16x+15$ ✗

$(x+3)(x+5) = x^2+8x+15$ ✓

$x^2 \quad -7x \quad +10$

BOTH ⊖ SAME SIGNS

$x^2 \quad -7x \quad +10$

x AND x −1 AND −10; −2 AND −5

$(x-1)(x-10) = x^2-11x+10$ ✗

$(x-2)(x-5) = x^2-7x+10$ ✓

$x^2 \quad +4x \quad -21$

DIFFERENT SIGNS

$x^2 \quad +4x \quad -21$

x AND x −1 AND +21; +1 AND −21; −3 AND +7; +3 AND −7

$(x-1)(x+21) = x^2+20x-21$ ✗

$(x+1)(x-21) = x^2-20x-21$ ✗

$(x-3)(x+7) = x^2+4x-21$ ✓

$(x+3)(x-7) = x^2-4x-21$ ✗

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Factorising Quadratics and Solving Equations

GENERAL II : THREE TERM ! (COEFFICIENT OF x^2 IS NOT ONE)

① $2x^2+7x+3$
$(2x+1)(x+3)$

② $3x^2+5x+2$
$(3x+2)(x+1)$

③ $2x^2+11x+14$
$(2x+7)(x+2)$

④ $5x^2-21x+4$
$(5x-1)(x-4)$

⑤ $3x^2-5x+2$
$(3x-2)(x-1)$

⑥ $6x^2-19x+10$
$(2x-5)(3x-2)$

⑦ $3x^2+13x-10$
$(3x-2)(x+5)$

⑧ $2x^2-13x-7$
$(2x+1)(x-7)$

⑨ $4x^2-4x-15$
$(2x+3)(2x-5)$

$2x^2 + 11x + 14$

BOTH ⊕ SAME SIGNS

$2x^2 + 11x + 14$

$2x$ AND x ; $+1$ AND $+14$; $+2$ AND $+7$

$(2x+1)(x+14) = 2x^2+29x+14$ ✗

$(2x+14)(x+1) = 2x^2+16x+14$ ✗

$(2x+2)(x+7) = 2x^2+16x+14$ ✗

$(2x+7)(x+2) = 2x^2+11x+14$ ✓

$5x^2 - 21x + 4$

BOTH ⊖ SAME SIGNS

$5x^2 - 21x + 4$

$5x$ AND x ; -1 AND -4 ; -2 AND -2

$(5x-1)(x-4) = 5x^2-21x+4$ ✓

$(5x-4)(x-1) = 5x^2-9x+4$ ✗

$(5x-2)(5x-2) = 5x^2-20x+4$ ✗

$2x^2 - 13x - 7$

DIFFERENT SIGNS

$2x^2 - 13x - 7$

$2x$ AND x ; $+1$ AND -7 ; -1 AND $+7$

$(2x+1)(x-7) = 2x^2-13x-7$ ✓

$(2x-7)(x+1) = 2x^2-5x-7$ ✗

$(2x-1)(x+7) = 2x^2+13x-7$ ✗

$(2x+7)(x-1) = 2x^2+5x-7$ ✗

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Factorising Quadratics and Solving Equations

BASIC III : GENERAL WITH SIMPLIFYING COMMON NUMERICAL FACTOR

① $2x^2 + 10x + 12$

$2(x+3)(x+2)$

② $6x^2 - 3x - 18$

$3(2x+3)(x-2)$

③ $5x^2 - 20$

$5(x+2)(x-2)$

$2x^2 + 10x + 12$

COMMON FACTOR

$2(x^2 + 5x + 6)$

NOW FACTORISE

$2(x+3)(x+2)$

$6x^2 - 3x - 18$

COMMON FACTOR

$3(2x^2 - x - 6)$

NOW FACTORISE

$3(2x+3)(x-2)$

$5x^2 - 20$

COMMON FACTOR

$5(x^2 - 4)$

NOW FACTORISE

DIFFERENCE BETWEEN TWO SQUARES

$5(x+2)(x-2)$

Factorising Quadratics and Solving Equations

Exercise A: Factorising Quadratics

Factorising quadratics means writing the **quadratic** as a **product** of **two linear factors** using **brackets**.

Basic I: Two Term! x as a Common Factor

Factorise these quadratics – they are a special case and can be done quickly.

(1) $x^2 + 2x$	(2) $x^2 - 5x$	(3) $x^2 + 8x$
(4) $2x^2 - 6x$	(5) $5x^2 + 20x$	(6) $6x^2 - 9x$

Basic II: Two Term! The Difference Between Two Squares

Factorise these quadratics – they are a special case and can be done quickly.

(7) $x^2 - 1$	(8) $x^2 - 36$	(9) $x^2 - 100$
(10) $4 - x^2$	(11) $9x^2 - 16$	(12) $25x^2 - 81y^2$

General I: Three Term! Coefficient of x^2 is One

Factorise these general quadratics, these are the basic general form.

(13) $x^2 + 7x + 10$	(14) $x^2 + 6x + 9$	(15) $x^2 + 7x + 6$
(16) $x^2 - 10x + 25$	(17) $x^2 - 9x + 14$	(18) $x^2 - 9x + 18$
(19) $x^2 + 2x - 8$	(20) $x^2 + x - 30$	(21) $x^2 - 7x - 18$

General II: Three Term! Coefficient of x^2 is Not One

Factorise these general quadratics, these are more complex.

(22) $2x^2 + 11x + 5$	(23) $5x^2 + 36x + 7$	(24) $2x^2 + 17x + 21$
(25) $2x^2 - 5x + 3$	(26) $3x^2 - 17x + 10$	(27) $5x^2 - 14x + 8$
(28) $2x^2 + 7x - 4$	(29) $3x^2 - 13x - 10$	(30) $4x^2 + 8x - 21$

Basic III: Three Term! General Type but with Common Numerical Factor

Factorise these quadratics – they are of the general form, but there is a common numerical factor throughout which when taken out simplifies the quadratic.

(31) $2x^2 + 16x + 30$	(32) $5x^2 + 10x - 15$	(33) $3x^2 - 12$

Factorising Quadratics and Solving Equations

Exercise A: Factorising Quadratics

Factorising quadratics means writing the **quadratic** as a **product** of **two linear factors** using **brackets**.

Mixed Quadratics

Factorise these quadratics.

(34) $x^2 + 6x - 27$	(35) $x^2 - 64$	(36) $x^2 - 12x + 35$
(37) $2x^2 + 5x + 2$	(38) $x^2 - 2x$	(39) $x^2 + 9x + 18$
(40) $4x^2 - 4x + 1$	(41) $3x^2 - 7x + 2$	(42) $4x^2 - 25$
(43) $4x^2 - 6x$	(44) $3x^2 - 3$	(45) $2x^2 + 8x - 42$
(46) $3x^2 - x - 10$	(47) $2x^2 + 5x - 7$	(48) $3x^2 + 7x + 2$
(49) $7x^2 - 33x - 10$	(50) $4x^2 - 40x + 100$	

Factorising Quadratics and Solving Equations

QUADRATIC EQUATIONS

Factorising can be used to solve a quadratic equation (providing the quadratic will factorise).

The equation needs to have the form quadratic = 0.

The solutions represent the roots of the quadratic i.e. the values of x where the curve cuts the x-axis.

EXAMPLE

Solve $x^2 - 7x + 10 = 0$

SOLUTION

$$x^2 - 7x + 10 = 0$$
$$(x-2)(x-5) = 0$$
$$x-2=0 \qquad x-5=0$$
$$x=2 \qquad x=5$$

EXAMPLE

Solve $x^2 - 3x - 10 = 0$

SOLUTION

$$x^2 - 3x - 10 = 0$$
$$(x+2)(x-5) = 0$$
$$x+2=0 \qquad x-5=0$$
$$x=-2 \qquad x=5$$

EXAMPLE

Solve $3x^2 - 5x + 2 = 0$

SOLUTION

$$3x^2 - 5x + 2 = 0$$
$$(3x-2)(x-1) = 0$$
$$3x-2=0 \qquad x-1=0$$
$$3x = 2 \qquad x = 1$$
$$x = \frac{2}{3}$$

EXAMPLE

Solve $4x^2 - 81 = 0$

SOLUTION

$$4x^2 - 81 = 0$$
$$(2x+9)(2x-9) = 0$$
$$2x+9=0 \qquad 2x-9=0$$
$$2x = -9 \qquad 2x = 9$$
$$x = -\frac{9}{2} \qquad x = \frac{9}{2}$$

EXAMPLE

Solve $x^2 + 5x = 0$

SOLUTION

$$x^2 + 5x = 0$$
$$x(x+5) = 0$$
$$x=0 \qquad x+5=0$$
$$x = -5$$

Factorising Quadratics and Solving Equations

Exercise B: Quadratic Equations

Factorising can be used to solve a **quadratic equation** (providing the quadratic will **factorise**).

The **equation** needs to have the form **quadratic = 0**.

The **solutions** represent the **roots** of the **quadratic** *i.e.* the values of x where the curve cuts the ***x*-axis**.

Solve these quadratic equations (they are questions 34-50 from the last exercise).

(1) $x^2+6x-27=0$	(2) $x^2-64=0$	(3) $x^2-12x+35=0$
(4) $2x^2+5x+2=0$	(5) $x^2-2x=0$	(6) $x^2+9x+18=0$
(7) $4x^2-4x+1=0$	(8) $3x^2-7x+2=0$	(9) $4x^2-25=0$
(10) $4x^2-6x=0$	(11) $3x^2-3=0$	(12) $2x^2+8x-42=0$
(13) $3x^2-x-10=0$	(14) $2x^2+5x-7=0$	(15) $3x^2+7x+2=0$
(16) $7x^2-33x-10=0$	(17) $4x^2-40x+100=0$	(18) $x^2+1=0$

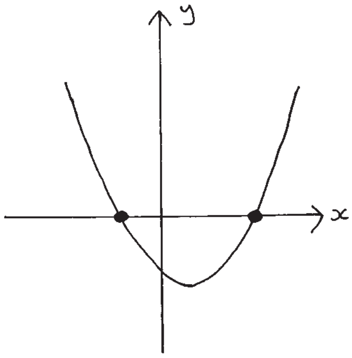
P. Leary

Factorising Quadratics and Solving Equations

QUADRATICS, EQUATIONS AND GRAPHS

The solutions of a quadratic equation of the form quadratic = 0 give key points on the quadratic graph.

The solutions represent the roots of the quadratic i.e. the values of x where the curve cuts the x-axis.



● = roots (where graph cuts the x-axis)

NOTE:

In the work in this unit only 'positive' U-shaped quadratics are considered.

EXAMPLE

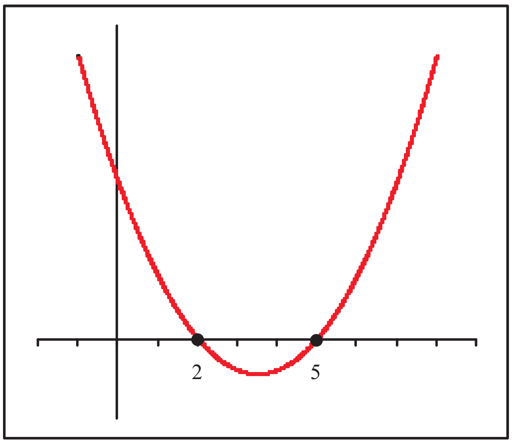
Find the roots of $y = x^2 - 7x + 10$ and sketch the graph of this quadratic.

SOLUTION

roots $x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$x = 2 \quad x = 5$



EXAMPLE

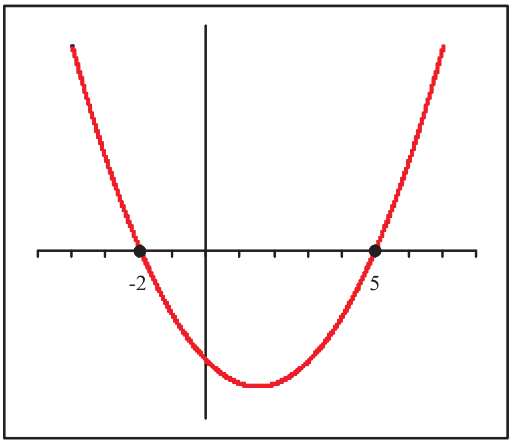
Find the roots of $y = x^2 - 3x - 10$ and sketch the graph of this quadratic.

SOLUTION

roots $x^2 - 3x - 10 = 0$

$(x+2)(x-5) = 0$

$x = -2 \quad x = 5$



Factorising Quadratics and Solving Equations

EXAMPLE

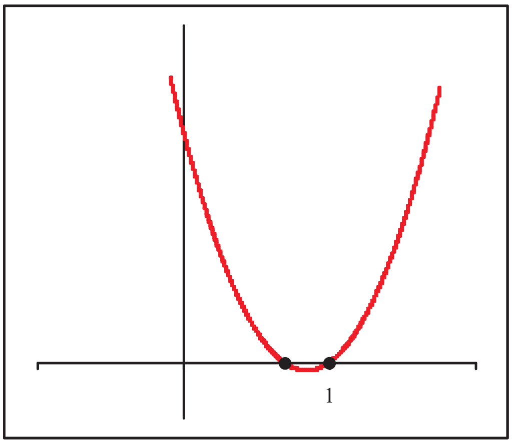
Find the roots of $y = 3x^2 - 5x + 2$ and sketch the graph of this quadratic.

SOLUTION

roots $3x^2 - 5x + 2 = 0$

$(3x-2)(x-1) = 0$

$x = \frac{2}{3} \quad x = 1$



EXAMPLE

Find the roots of $y = 4x^2 - 81$ and sketch the graph of this quadratic.

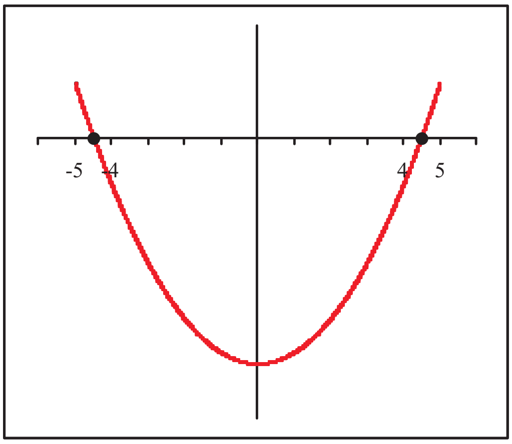
SOLUTION

roots $4x^2 - 81 = 0$

$(2x+9)(2x-9) = 0$

$x = -\frac{9}{2} \quad x = \frac{9}{2}$

$x = -4\frac{1}{2} \quad x = 4\frac{1}{2}$



EXAMPLE

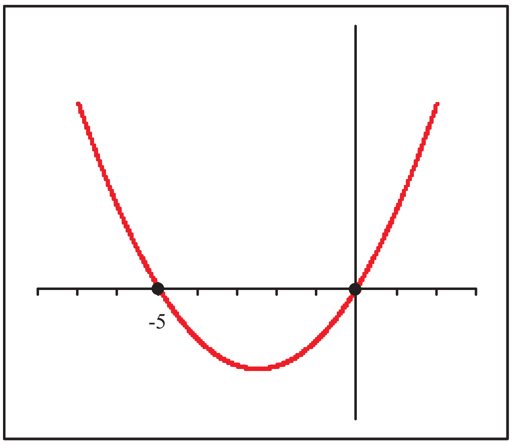
Find the roots of $y = x^2 + 5x$ and sketch the graph of this quadratic.

SOLUTION

roots $x^2 + 5x = 0$

$x(x+5) = 0$

$x = 0 \quad x = -5$



Exercise C: Factorising and Graphs

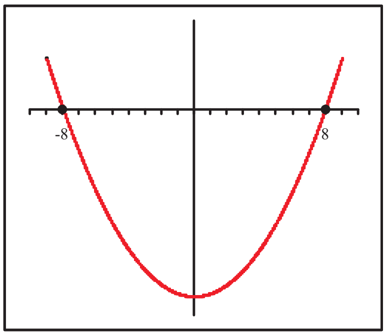
Factorising can be used to solve a **quadratic equation** (providing the quadratic will **factorise**).

The **equation** needs to have the form **quadratic = 0**.

The **solutions** represent the **roots** of the **quadratic** *i.e.* the values of x where the curve cuts the ***x*-axis**.

Match these quadratic graphs to the quadratics/equations from the last exercise.

(1)



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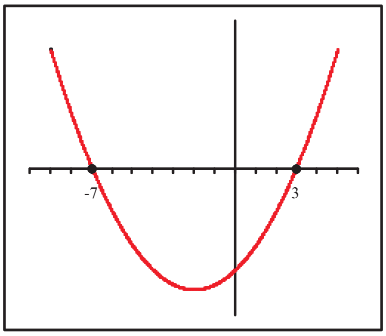

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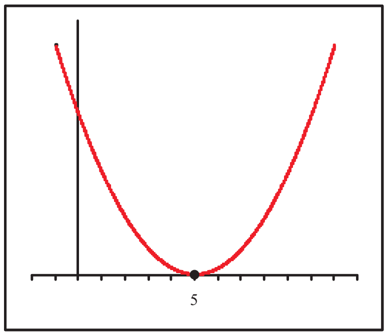

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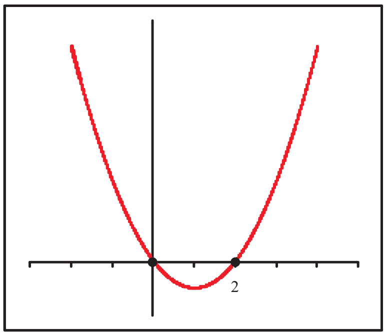

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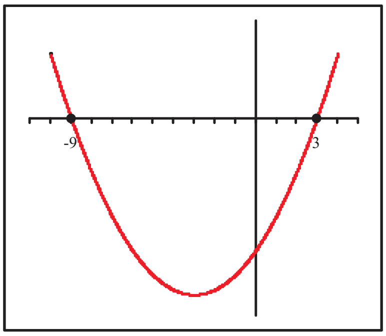

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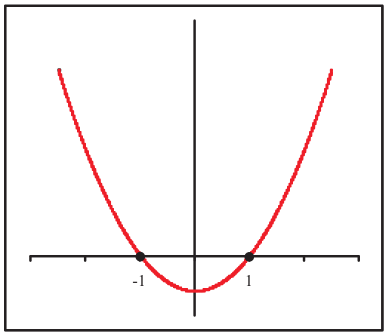

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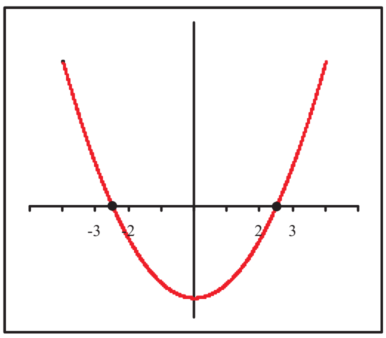

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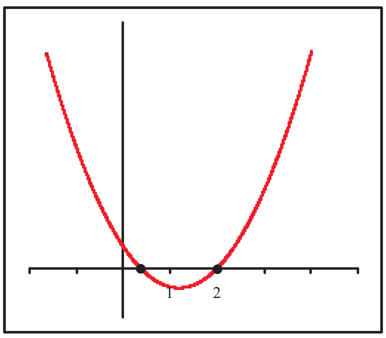

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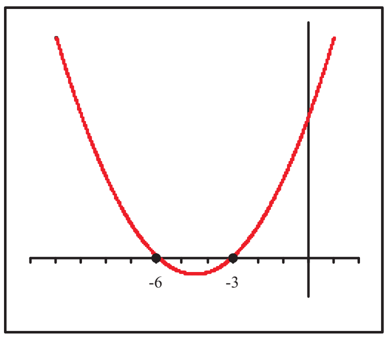


Factorising Quadratics and Solving Equations

Exercise C: Factorising and Graphs

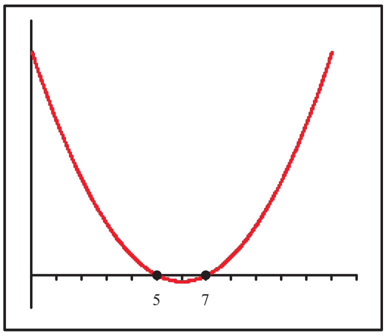
Factorising can be used to solve a **quadratic equation** (providing the quadratic will **factorise**).

The **equation** needs to have the form **quadratic = 0**.

The **solutions** represent the **roots** of the **quadratic** *i.e.* the values of x where the curve cuts the ***x*-axis**.

Match these quadratic graphs to the quadratics/equations from the last exercise.

(10)



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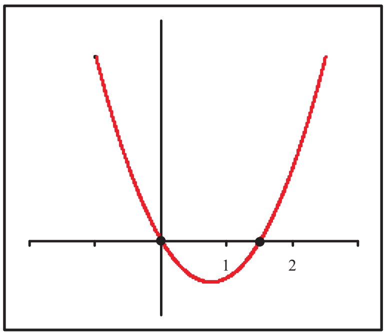

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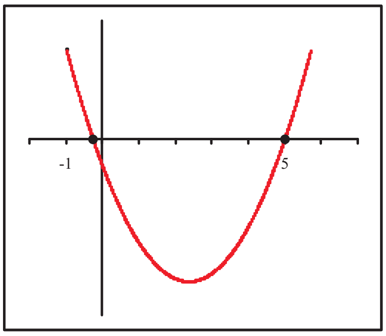

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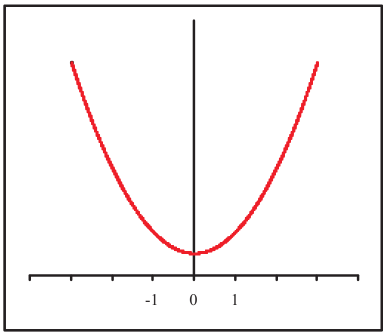

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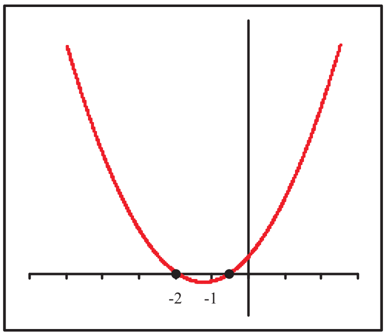

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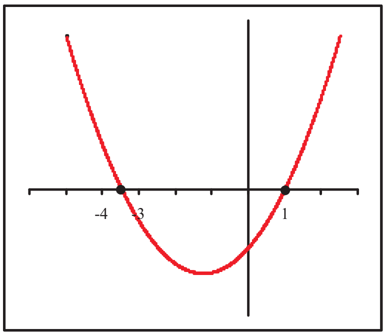

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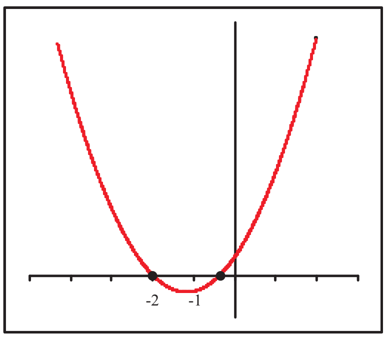

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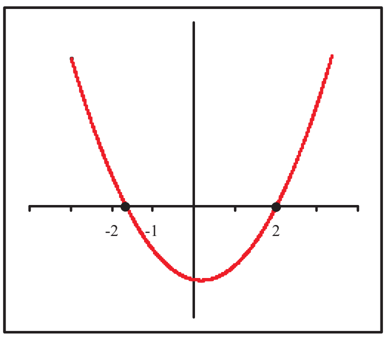

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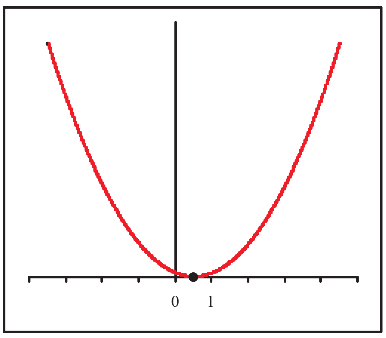


FACTORISING QUADRATICS EXERCISE A

BASIC I : TWO TERM ! x AS A COMMON FACTOR

① x^2+2x
$x(x+2)$

② x^2-5x
$x(x-5)$

③ x^2+8x
$x(x+8)$

④ $2x^2-6x$
$2x(x-3)$

⑤ $5x^2+20x$
$5x(x+4)$

⑥ $6x^2-9x$
$3x(2x-3)$

BASIC II : TWO TERM ! DIFFERENCE BETWEEN TWO SQUARES

⑦ x^2-1
$(x+1)(x-1)$

⑧ x^2-36
$(x+6)(x-6)$

⑨ x^2-100
$(x+10)(x-10)$

⑩ $4-x^2$
$(2+x)(2-x)$

⑪ $9x^2-16$
$(3x+4)(3x-4)$

⑫ $25x^2-81y^2$
$(5x+9y)(5x-9y)$

GENERAL I : THREE TERM ! COEFFICIENT OF x^2 IS ONE

⑬ $x^2+7x+10$
$(x+2)(x+5)$

⑭ x^2+6x+9
$(x+3)(x+3)$

⑮ x^2+7x+6
$(x+6)(x+1)$

⑯ $x^2-10x+25$
$(x-5)(x-5)$

⑰ $x^2-9x+14$
$(x-2)(x-7)$

⑱ $x^2-9x-18$
$(x-3)(x-6)$

⑲ x^2+2x-8
$(x+4)(x-2)$

⑳ x^2+x-30
$(x+6)(x-5)$

㉑ $x^2-7x-18$
$(x+2)(x-9)$

GENERAL II : THREE TERM ! COEFFICIENT OF x^2 IS NOT ONE

㉒ $2x^2+11x+5$
$(2x+1)(x+5)$

㉓ $5x^2+36x+7$
$(5x+1)(x+7)$

㉔ $2x^2+17x+21$
$(2x+3)(x+7)$

㉕ $2x^2-5x+3$
$(2x-3)(x-1)$

㉖ $3x^2-17x+10$
$(3x-2)(x-5)$

㉗ $5x^2-14x+8$
$(5x-4)(x-2)$

㉘ $2x^2+7x-4$
$(2x-1)(x+4)$

㉙ $3x^2-13x-10$
$(3x+2)(x-5)$

㉚ $4x^2+8x-21$
$(2x+7)(2x-3)$

BASIC III : GENERAL WITH SIMPLIFYING COMMON NUMERICAL FACTOR

㉛ $2x^2+16x+30$
$2(x+3)(x+5)$

㉜ $5x^2+10x-15$
$5(x+3)(x-1)$

㉝ $3x^2-12$
$3(x+2)(x-2)$

FACTORISING QUADRATICS EXERCISE A

MIXED QUESTIONS

(34) $x^2+6x-27$
$(x-3)(x+9)$

(35) x^2-64
$(x+8)(x-8)$

(36) $x^2-12x+35$
$(x-5)(x-7)$

(37) $2x^2+5x+2$
$(2x+1)(x+2)$

(38) x^2-2x
$(x(x-2)$

(39) $x^2+9x+18$
$(x+3)(x+6)$

(40) $4x^2-4x+1$
$(2x-1)(2x-1)$

(41) $3x^2-7x+2$
$(3x-1)(x-2)$

(42) $4x^2-25$
$(2x+5)(2x-5)$

(43) $4x^2-6x$
$2x(2x-3)$

(44) $3x^2-3$
$3(x+1)(x-1)$

(45) $2x^2+8x-42$
$2(x-3)(x+7)$

(46) $3x^2-x-10$
$(3x+5)(x-2)$

(47) $2x^2+5x-7$
$(2x+7)(x-1)$

(48) $3x^2+7x+2$
$(3x+1)(x+2)$

(49) $7x^2-33x-10$
$(7x+2)(x-5)$

(50) $4x^2-40x+100$
$4(x-5)(x-5)$

QUADRATIC EQUATIONS EXERCISE B

(1) $x=-9, 3$

(2) $x=-8, 8$

(3) $x=5, 7$

(4) $x=-2, -1/2$

(5) $x=0, 2$

(6) $x=-6, -3$

(7) $x=1/2$

(8) $x=1/3, 2$

(9) $x=-5/2, 5/2$

(10) $x=0, 3/2$

(11) $x=-1, 1$

(12) $x=-7, 3$

(13) $x=-5/3 \quad x=2$

(14) $x=-7/2, 1$

(15) $x=-1/3, -2$

(16) $x=-2/7, 5$

(17) $x=5$

(18) NO SOLUTION

$\uparrow$

$x^2+1=0$

FACTORISING QUADRATICS EXERCISE C

1. $y = x^2 - 64$
2. $y = 2x^2 + 8x - 42$
3. $y = 4x^2 - 40x + 100$
4. $y = x^2 - 2x$
5. $y = x^2 + 6x - 27$
6. $y = 3x^2 - 3$
7. $y = 4x^2 - 25$
8. $y = 3x^2 - 7x + 2$
9. $y = x^2 + 9x + 18$
10. $y = x^2 - 12x + 35$
11. $y = 4x^2 - 6x$
12. $y = 7x^2 - 33x - 10$
13. $y = x^2 + 1$
14. $y = 2x^2 + 5x + 2$
15. $y = 2x^2 + 5x - 7$
16. $y = 3x^2 + 7x + 2$
17. $y = 3x^2 - x - 10$
18. $y = 4x^2 - 4x + 1$

SIMULTANEOUS EQUATIONS

Non-Linear Simultaneous Equations

Objectives

★ know basic algebraic skills

- ☆ expand brackets
- ☆ rearrange basic formulae
- ☆ substitute numbers into algebraic expressions

★ solve quadratic equations by factorising or otherwise

★ appreciate that when you solve equations simultaneously, you find where their graphs cross

★ solve simultaneous equations by substitution

★ solve simultaneous equations involving a line and a quadratic (to determine their crossing points)

★ solve simultaneous equations involving a line and a circle (to determine their crossing points)

★ obtain the y-values (know the line is easiest)

★ verify solutions by substitution

REFERENCE IN OTHER RESOURCES

Head Start: Topic not covered.

Alpha Workbooks: Topic not covered.

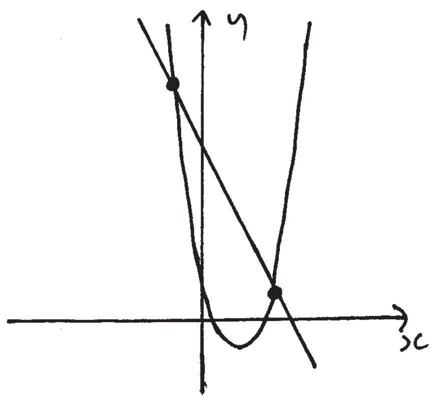
Simultaneous Equations - Non-Linear Simultaneous Equations

INTERSECTIONS BETWEEN LINES AND CURVES

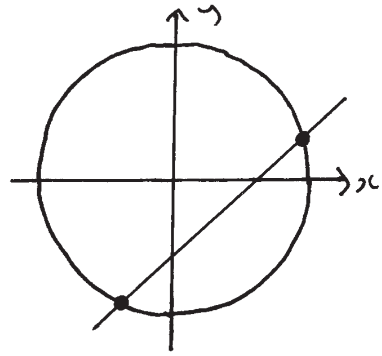
NON-LINEAR SIMULTANEOUS EQUATIONS

Suppose you want to calculate the crossing points of a line and a curve.

e.g. 1 LINE AND QUADRATIC



e.g. 2 LINE AND CIRCLE



Solving the equations simultaneously will find the crossing point.

The method used will be substitution. Substitute the equation in the form $y = \ldots\ldots$ into the other equation.

The resulting equation should be a quadratic equation (in x) which can be solved by factorising in most cases. If the quadratic does not factorise use the quadratic formula or complete the square.

The y-values of the crossing point are also often required. To find these it is easiest to put your x-values found into the straight line. You could then check in the curve if desired.

Simultaneous Equations - Non-Linear Simultaneous Equations

Quadratics and Lines

EXAMPLE

Find the points of intersection between :

$y = x^2 + 4x - 11$ ① (quadratic curve)

$y = 2x - 3$ ② (straight line)

SOLUTION

$y = x^2 + 4x - 11$ ①

$y = 2x - 3$ ②

SUBSTITUTE ① IN ② (EQUATE)

$x^2 + 4x - 11 = 2x - 3$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4 \qquad x = 2$

* $y = -11 \qquad y = 1$

OBTAINING y-VALUES *

To obtain the y-values put the x-values found into either equation.

It is much easier to put them into the line ②

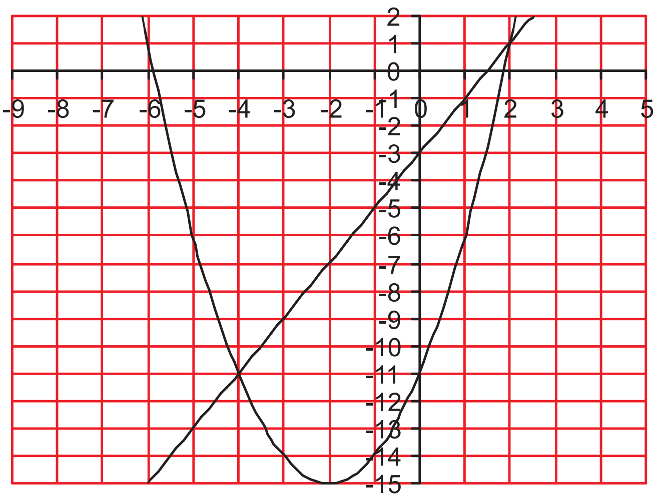
$y = 2(-4) - 3 = -8 - 3 = -11$

$y = 2(2) - 3 = 4 - 3 = 1$

This will do!
The quadratic would verify.

$y = (-4)^2 + 4(-4) - 11 = 16 - 16 - 11 = -11$

$y = (2)^2 + 4(2) - 11 = 4 + 8 - 11 = 1$



EXAMPLE

Solve the following equations simultaneously :

$y = 4x^2 - x - 7$ ① (quadratic curve)

$y = 2 - x$ ② (straight line)

SOLUTION

$y = 4x^2 - x - 7$ ①

$y = 2 - x$ ②

SUBSTITUTE ① IN ②

$4x^2 - x - 7 = 2 - x$

$4x^2 - 9 = 0$

$(2x+3)(2x-3) = 0$

$x = -\frac{3}{2} \qquad x = \frac{3}{2}$

$y = \frac{7}{2} \qquad y = \frac{1}{2}$

OBTAINING y-VALUES *

To obtain the y-values put the x-values found into either equation.

It is much easier to put them into the line ②

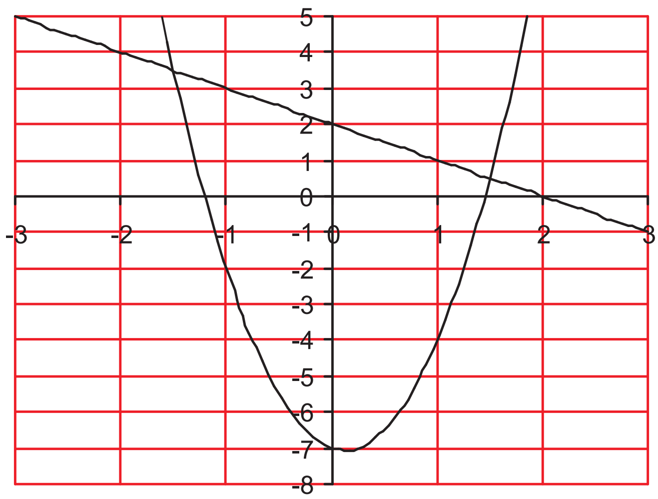
$y = 2 - (-\frac{3}{2}) = 2 + \frac{3}{2} = 3\frac{1}{2} = \frac{7}{2}$

$y = 2 - \frac{3}{2} = \frac{1}{2}$

This will do!
The quadratic would verify

$y = 4(-\frac{3}{2})^2 - (-\frac{3}{2}) - 7 = 9 + 1\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$

$y = 4(\frac{3}{2})^2 - (\frac{3}{2}) - 7 = 9 - 1\frac{1}{2} - 7 = \frac{1}{2}$



Simultaneous Equations - Non-Linear Simultaneous Equations

Exercise A: Quadratics and Lines

Find the **points of intersection** of the **two graphs** (**quadratic** and **line**).

To do this, **solve the equations simultaneously** using the **substitution method**.

You should always end up solving a **quadratic equation**.

Find the points of intersection between the given curve and the line giving both the x and the y coordinates of the solutions.

(1) $y = x^2 + 3x - 3$
$y = 2x - 1$

(2) $y = x^2 - 5x + 10$
$y = 3x - 5$

(3) $y = x^2 + 5$
$y = 4 - 2x$

(4) $y = 2x^2 - 8x - 3$
$y = x + 2$

(5) $y = 2x^2 - 7x + 4$
$y = 1 - 2x$

(6)* $y = 2x^2 - 15x - 7$
$y = 5x - x^2$

(7) $y = 4x^2 + 7x - 30$
$y = 7x - 5$

(8) $y = 2x^2 - 5x + 3$
$y = x + 3$

*This question is finding the intersection between two quadratic graphs, not a line and a curve.

Simultaneous Equations - Non-Linear Simultaneous Equations

Quadratics and Lines

EXAMPLE

Solve these equations simultaneously:

$y = x^2 - 4x$ ① (quadratic curve)

$2x + y = 3$ ② (straight line)

SOLUTION

$y = x^2 - 4x$ ①

$2x + y = 3$ ②

SUBSTITUTE ① IN ②

$2x + (x^2 - 4x) = 3$

$2x + x^2 - 4x = 3$

$x^2 - 2x = -3$

$x^2 - 2x + 3 = 0$

$(x+1)(x-3) = 0$

$x = -1 \quad x = 3$

* $y = 5 \quad y = -3$

OBTAINING y-VALUES *

To obtain the y-values put the x-values found into either equation.

It is much easier to put them into the line ②

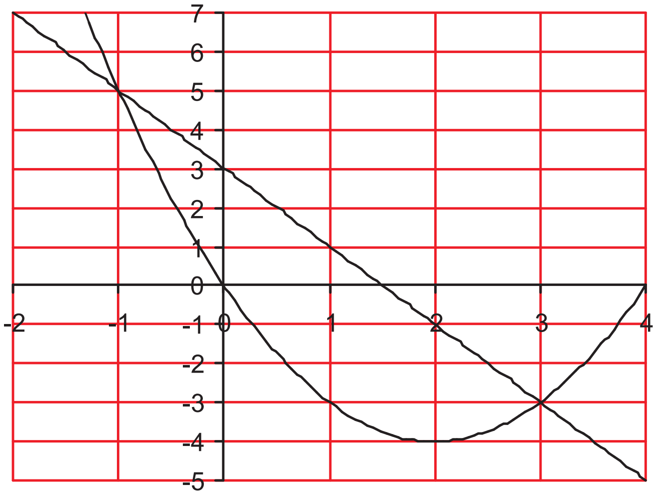
$-2 + y = 3 \quad y = 5$

$6 + y = 3 \quad y = -3$

This will do!
The quadratic would verify.

$y = (-1)^2 - 4(-1) = 1 + 4 = 5$

$y = (3)^2 - 4(3) = 9 - 12 = -3$



EXAMPLE

Find the points of intersection of the line and curve:

$3x + 2y = 14$ ① (straight line)

$y = x^2 - 3x + 7$ ② (quadratic curve)

SOLUTION

$3x + 2y = 14$ ①

$y = x^2 - 3x + 7$ ②

SUBSTITUTE ② IN ①

$3x + 2(x^2 - 3x + 7) = 14$

$3x + 2x^2 - 6x + 14 = 14$

$2x^2 - 3x + 14 = 14$

$2x^2 - 3x = 0$

$x(2x - 3) = 0$

$x = 0 \quad x = \frac{3}{2}$

* $y = 7 \quad y = \frac{19}{4}$

OBTAINING y-VALUES *

To obtain the y-values put the x-values found into either equation.

It is much easier to put them into the line ①

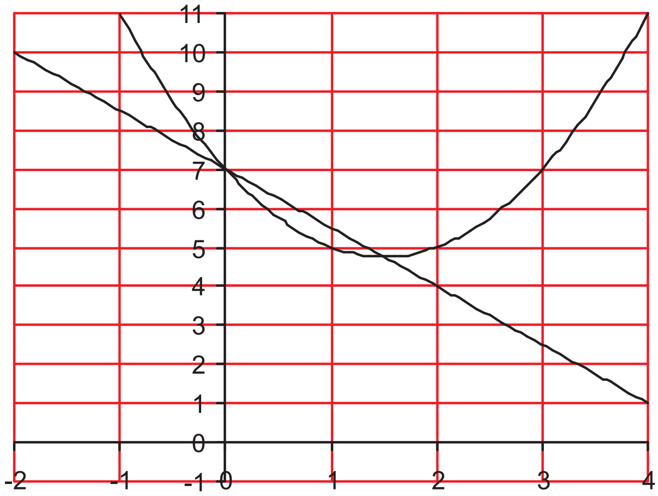
$0 + 2y = 14 \quad y = 7$

$\frac{9}{2} + 2y = 14$

$\rightarrow 2y = \frac{19}{2}$

$y = \frac{19}{4}$

The quadratic would verify.



Simultaneous Equations - Non-Linear Simultaneous Equations

Exercise B: Quadratics and Lines

Find the **points of intersection** of the **two graphs** (**quadratic** and **line**).

To do this, **solve the equations simultaneously** using the **substitution method**.

You should always end up solving a **quadratic equation**.

Find the points of intersection between the given curve and the line giving both the x and the y coordinates of the solutions.

(1) $y = x^2 - 3x - 20$
$x + y = 4$

(2) $y = x^2 - 2x - 10$
$2x + y = 6$

(3) $y = x^2 - 2x - 3$
$2x + y = -2$

(4) $y = 2x^2 - 4x - 7$
$2x + y = 5$

(5) $y = x^2 - 4x$
$x + 2y = 4$

(6) $7x + 2y = 14$
$y = x^2 - 7x + 7$

Simultaneous Equations - Non-Linear Simultaneous Equations

Circles and Lines

EXAMPLE

Find where the line crosses the curve.

$x^2 + y^2 = 16$ ① (circle radius 4 centre (0,0))
$y = x - 4$ ② (straight line)

SOLUTION

$x^2 + y^2 = 16$ ①
$y = x - 4$ ②

SUBSTITUTE ② IN ①

$x^2 + (x-4)^2 = 16$

$x^2 + x^2 - 8x + 16 = 16$

$2x^2 - 8x + 16 = 16$

$2x^2 - 8x = 0$

$2x(x-4) = 0$

$x = 0 \qquad x = 4$

* $y = -4 \qquad y = 0$

OBTAINING y-VALUES *

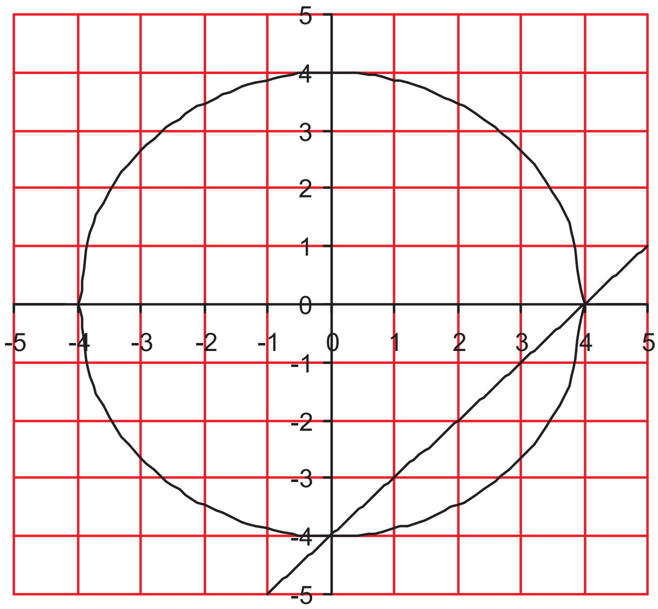
To obtain the y-values put the x-values found into either equation.

It is much easier to put them into the line ②

$y = 0 - 4 = -4$

$y = 4 - 4 = 0$

Substitution into the circle would verify.



EXAMPLE

Solve these equations simultaneously:

$x^2 + y^2 = 29$ ① (circle radius $\sqrt{29}$ centre (0,0))
$y = x - 3$ ② (straight line)

SOLUTION

$x^2 + y^2 = 29$ ①
$y = x - 3$ ②

SUBSTITUTE ② IN ①

$x^2 + (x-3)^2 = 29$

$x^2 + x^2 - 6x + 9 = 29$

$2x^2 - 6x + 9 = 29$

$2x^2 - 6x - 20 = 0$

$2(x^2 - 3x - 10) = 0$

$2(x+2)(x-5) = 0$

$x = -2 \qquad x = 5$

$y = -5 \qquad y = 2$

OBTAINING y-VALUES *

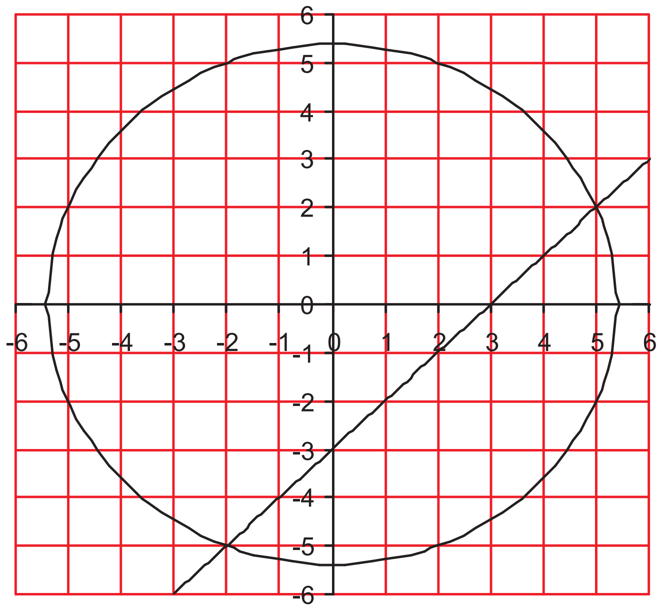
To obtain the y-values put the x-values found into either equation.

It is much easier to put them into the line ②

$y = -2 - 3 = -5$

$y = 5 - 3 = 2$

Substitution into the circle would verify.



Simultaneous Equations - Non-Linear Simultaneous Equations

Exercise C: Circles and Lines

Find the **points of intersection** of the **two graphs** (**circle** and **line**).

To do this, **solve the equations simultaneously** using the **substitution method**.

You should always end up solving a **quadratic equation**.

Find the points of intersection between the given curve and the line giving both the x and the y coordinates of the solutions.

(1) $x^2 + y^2 = 100$
$y = 3x + 10$

(2) $x^2 + y^2 = 25$
$y = x - 7$

(3) $x^2 + y^2 = 17$
$y = 4x$

(4) $x^2 + y^2 = 20$
$y = 10 - 2x$

(5) $x^2 + y^2 = 10$
$y = 3x + 10$

(6) $x^2 + y^2 = 169$
$2y = 3x - 39$

LINES AND CURVES

EXERCISE 1

① $y = x^2 + 3x - 3$ ①
$y = 2x - 1$ ②

SUBSTITUTE ① IN ②

$x^2 + 3x - 3 = 2x - 1$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad x = 1$

$y = -5 \quad y = 1$

② $y = x^2 - 5x + 10$ ①
$y = 3x - 5$ ②

SUBSTITUTE ① IN ②

$x^2 - 5x + 10 = 3x - 5$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3 \quad x = 5$

$y = 4 \quad y = 10$

③ $y = x^2 + 5$ ①
$y = 4 - 2x$ ②

SUBSTITUTE ① IN ②

$x^2 + 5 = 4 - 2x$

$x^2 + 2x + 1 = 0$

$(x+1)(x+1) = 0$

$x = -1$

$y = 6$

④ $y = 2x^2 - 8x - 3$
$y = x + 2$

SUBSTITUTE ① IN ②

$2x^2 - 8x - 3 = x + 2$

$2x^2 - 9x - 5 = 0$

$(2x+1)(x-5) = 0$

$x = -\frac{1}{2} \quad x = 5$

$y = \frac{3}{2} \quad y = 7$

LINES AND CURVES EXERCISE 1

⑤ $y = 2x^2 - 7x + 4$ ①
$y = 1 - 2x$ ②

SUBSTITUTE ① IN ②

$2x^2 - 7x + 4 = 1 - 2x$

$2x^2 - 5x + 3 = 0$

$(2x-3)(x-1) = 0$

$x = \frac{3}{2}$ $x = 1$

$y = -2$ $y = -1$

⑥ $y = 2x^2 - 15x - 7$ ①
$y = 5x - x^2$ ②

SUBSTITUTE ① IN ②

$2x^2 - 15x - 7 = 5x - x^2$

$3x^2 - 20x - 7 = 0$

$(3x+1)(x-7) = 0$

$x = -\frac{1}{3}$ $x = 7$

PUT VALUES IN ②

$y = 5(-\frac{1}{3}) - (-\frac{1}{3})^2$ $y = 5(7) - (7)^2$

$y = -\frac{5}{3} - \frac{1}{9}$ $y = 35 - 49$

$y = -\frac{15}{9} - \frac{1}{9}$ $y = -14$

$y = -\frac{16}{9}$

⑦ $y = 4x^2 + 7x - 30$ ①
$y = 7x - 5$ ②

SUBSTITUTE ① IN ②

$4x^2 + 7x - 30 = 7x - 5$

$4x^2 - 25 = 0$

$(2x+5)(2x-5) = 0$

$x = -\frac{5}{2}$ $x = \frac{5}{2}$

$y = -\frac{45}{2}$ $y = \frac{25}{2}$

⑧ $y = 2x^2 - 5x + 3$ ①
$y = x + 3$ ②

SUBSTITUTE ① IN ②

$2x^2 - 5x + 3 = x + 3$

$2x^2 - 6x = 0$

$2x(x-3) = 0$

$x = 0$ $x = 3$

$y = 3$ $y = 6$

LINES AND CURVES

EXERCISE 2

① $y = x^2 - 3x - 20$ ①
$x + y = 4$ ②

SUBSTITUTE ① IN ②

$x + (x^2 - 3x - 20) = 4$

$x + x^2 - 3x - 20 = 4$

$x^2 - 2x - 20 = 4$

$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$x = 6 \qquad x = -4$

$y = -2 \qquad y = 8$

② $y = x^2 - 2x - 10$ ①
$2x + y = 6$ ②

SUBSTITUTE ① IN ②

$2x + (x^2 - 2x - 10) = 6$

$2x + x^2 - 2x - 10 = 6$

$x^2 - 10 = 6$

$x^2 - 16 = 0$

$(x-4)(x+4) = 0$

$x = 4 \qquad x = -4$

$y = -2 \qquad y = 14$

③ $y = x^2 - 2x - 3$ ①
$2x + y = -2$ ②

SUBSTITUTE ① IN ②

$2x + (x^2 - 2x - 3) = -2$

$2x + x^2 - 2x - 3 = -2$

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1 \qquad x = 1$

$y = 0 \qquad y = -4$

④ $y = 2x^2 - 4x - 7$ ①
$2x + y = 5$ ②

SUBSTITUTE ① IN ②

$2x + (2x^2 - 4x - 7) = 5$

$2x + 2x^2 - 4x - 7 = 5$

$2x^2 - 2x - 7 = 5$

$2x^2 - 2x - 12 = 0$

$2(x^2 - x - 6) = 0$

$2(x+2)(x-3) = 0$

$x = -2 \qquad x = 3$

$y = 9 \qquad y = -1$

LINES AND CURVES

EXERCISE 2

(5) $y = x^2 - 4x$ (1)
$x + 2y = 4$ (2)

SUBSTITUTE (1) IN (2)

$x + 2(x^2 - 4x) = 4$

$x + 2x^2 - 8x = 4$

$2x^2 - 7x = 4$

$2x^2 - 7x - 4 = 0$

$(2x+1)(x-4) = 0$

$x = -\frac{1}{2}$ $\quad x = 4$

$y = \frac{9}{4}$ $\quad y = 0$

(6) $7x + 2y = 14$ (1)
$y = x^2 - 7x + 7$ (2)

SUBSTITUTE (2) IN (1)

$7x + 2(x^2 - 7x + 7) = 14$

$7x + 2x^2 - 14x + 14 = 14$

$2x^2 - 7x + 14 = 14$

$2x^2 - 7x = 0$

$x(2x - 7) = 0$

$x = 0$ $\quad x = \frac{7}{2}$

$y = 7$ $\quad y = \frac{-21}{4}$

LINES AND CIRCLES EXERCISE 3

① $x^2+y^2 = 100$ ①
$y = 3x+10$ ②

SUBSTITUTE ② IN ①

$x^2+(3x+10)^2 = 100$

$x^2+9x^2+60x+100 = 100$

$10x^2+60x+100 = 100$

$10x^2+60x = 0$

$10x(x+6) = 0$

$x=0 \qquad x=-6$

$y=10 \qquad y=-8$

② $x^2+y^2 = 25$ ①
$y = x-7$ ②

SUBSTITUTE ② IN ①

$x^2+(x-7)^2 = 25$

$x^2+x^2-14x+49 = 25$

$2x^2-14x+49 = 25$

$2x^2-14x+24 = 0$

$2(x^2-7x+12) = 0$

$2(x-3)(x-4) = 0$

$x=3 \qquad x=4$

$y=-4 \qquad y=-3$

③ $x^2+y^2 = 17$ ①
$y = 4x$ ②

SUBSTITUTE ② IN ①

$x^2+(4x)^2 = 17$

$x^2+16x^2 = 17$

$17x^2 = 17$

$17x^2-17 = 0$

$17(x^2-1) = 0$

$17(x+1)(x-1) = 0$

$x=-1 \qquad x=1$

$y=-4 \qquad y=4$

④ $x^2+y^2 = 20$ ①
$y = 10-2x$ ②

SUBSTITUTE ② IN ①

$x^2+(10-2x)^2 = 20$

$x^2+100-40x+4x^2 = 20$

$5x^2-40x+100 = 20$

$5x^2-40x+80 = 0$

$5(x^2-8x+16) = 0$

$5(x-4)(x-4) = 0$

$x=4$

$y=2$

TANGENT!

LINES AND CIRCLES EXERCISE 3

(5) $x^2 + y^2 = 10$ (1)

$y = 3x + 10$ (2)

SUBSTITUTE (2) IN (1)

$x^2 + (3x+10)^2 = 10$

$x^2 + 9x^2 + 60x + 100 = 10$

$10x^2 + 60x + 100 = 10$

$10x^2 + 60x + 90 = 0$

$10(x^2 + 6x + 9) = 0$

$10(x+3)(x+3) = 0$

$x = -3$

$y = 1$ TANGENT!!

(6) $x^2 + y^2 = 169$ (1)

$2y = 3x - 39$ (2)

SUBSTITUTE REARRANGED (2) IN (1)

$2y = 3x - 39 \rightarrow y = \frac{3x-39}{2}$

$x^2 + \left(\frac{3x-39}{2}\right)^2 = 169$

$x^2 + \frac{9x^2 - 234x + 1521}{4} = 169$

CLEAR FRACTION

$4 \times x^2 + 9x^2 - 234x + 1521 = 4 \times 169$

$4x^2 + 9x^2 - 234x + 1521 = 676$

$13x^2 - 234x + 845 = 0$

$13(x^2 - 18x + 65) = 0$

$13(x - 5)(x - 13) = 0$

$x = 5$ $x = 13$

$y = -12$ $y = 0$

QUADRATICS

Completing the Square

Objectives

★ be able to complete the square for basic quadratics

★ solve quadratic equations by completing the square

★ be aware of different ways of solving quadratic equations

★ know how the completed square form relates to the graph (how to get the vertex)

★ match quadratics to their graphs and vice-versa through key features

REFERENCE IN OTHER RESOURCES

Head Start: Topic not covered.

Alpha Workbooks: Topic not covered.

Quadratics - Completing the Square

COMPLETING THE SQUARE

Completing the square is another way of writing a quadratic

The following table gives some quadratics and their completed square form

QUADRATIC	COMPLETED SQUARE
x^2-4x-5	$(x-2)^2-9$
$x^2+8x+15$	$(x+4)^2-1$
x^2-2x+6	$(x-1)^2+5$
$2x^2-4x-16$	$2(x-1)^2-18$

ALGEBRAIC METHOD

EXAMPLE

Write x^2-4x-5 in completed square form.

SOLUTION

$$x^2-4x-5$$

↓ half ↓ subtract square of number in bracket

$$(x-2)^2-5\ -4$$

$$(x-2)^2-9$$

EXAMPLE

Write x^2-2x+6 in the form $(x+a)^2+b$

i.e. complete the square.

SOLUTION

$$x^2-2x+6$$

↓ half ↓ subtract square of number in bracket

$$(x-1)^2+6\ -1$$

$$(x-1)^2+5$$

SOLVING QUADRATIC EQUATIONS

Completing the square can be used to solve quadratic equations.

Alternative methods are:
- factorising
- quadratic formula

EXAMPLE

Solve $x^2-4x-5=0$ by completing the square.

SOLUTION

$$x^2-4x-5=0$$
$$(x-2)^2-9=0$$
$$(x-2)^2=9$$
$$x-2=\pm 3$$
$$x=2\pm 3 \rightarrow x=5,-1$$

EXAMPLE

Solve $x^2+8x+15=0$ by completing the square.

SOLUTION

$$x^2+8x+15=0$$
$$(x+4)^2-1=0$$
$$(x+4)^2=1$$
$$x+4=\pm 1$$
$$x=-4\pm 1 \rightarrow x=-5,-3$$

EXAMPLE

Solve $x^2+2x-5=0$

SOLUTION

$$x^2+2x-5$$

complete the square

$$(x+1)^2-5-1$$
$$(x+1)^2-6$$

so

$$(x+1)^2-6=0$$
$$(x+1)^2=6$$
$$x+1=\pm\sqrt{6}$$
$$x=-1\pm\sqrt{6}$$

Quadratics - Completing the Square

Exercise A: Algebraic Method

Completing the square is another way of writing a **quadratic**.

The completed square form is useful as it gives you information about where the **bottom** (or **top**) of a **quadratic graph** is.

Completing the square can also be used to **solve quadratic equations**.

Complete the square for these basic quadratics.
All of these quadratics will factorise.

(1) $x^2 - 6x + 8$ (2) $x^2 + 2x - 3$

(3) $x^2 + 12x + 20$ (4) $x^2 + 4x$

(5) $x^2 - 2x - 63$ (6) $x^2 - 2x - 15$

(7) $x^2 - 6x$ (8) $x^2 + 4x - 21$

(9) $x^2 - 6x - 7$ (10) $x^2 - 2x$

(11) $x^2 - 10x + 21$ (12) $x^2 + 6x - 27$

(13) $x^2 - 4x + 3$ (14) $x^2 - 8x - 9$

Complete the square for these basic quadratics.
These quadratics will not factorise but some do have roots.

(15) $x^2 - 2x - 2$ (16) $x^2 + 4x - 3$

(17) $x^2 - 6x + 13$ (18) $x^2 + 10x + 30$

Complete the square for these harder quadratics.
The two quadratics here will factorise (they are a special case!).

(19) $2x^2 - 12x + 16$ (20) $3x^2 - 12x - 63$

Quadratics - Completing the Square

Exercise B: Solving Quadratic Equations by Completing the Square

Completing the square can be used to **solve quadratic equations**.

Solving the equation will find the **roots** of the **quadratic graph**.

The **roots** of a graph are where they **cross the *x*-axis**.

Solve the following equations by completing the square.
Check your answer by factorising.

(1) $x^2 - 6x + 8 = 0$
$(x-3)^2 - 1 = 0$

(2) $x^2 + 2x - 3 = 0$
$(x+1)^2 - 4 = 0$

(3) $x^2 + 12x + 20 = 0$
$(x+6)^2 - 16 = 0$

(4) $x^2 + 4x = 0$
$(x+2)^2 - 4 = 0$

(5) $x^2 - 2x - 63 = 0$
$(x-1)^2 - 64 = 0$

(6) $x^2 - 2x - 15 = 0$
$(x-1)^2 - 16 = 0$

(7) $x^2 - 6x = 0$
$(x-3)^2 - 9 = 0$

(8) $x^2 + 4x - 21 = 0$
$(x+2)^2 - 25 = 0$

(9) $x^2 - 6x - 7 = 0$
$(x-3)^2 - 16 = 0$

(10) $x^2 - 2x = 0$
$(x-1)^2 - 1 = 0$

(11) $x^2 - 10x + 21 = 0$
$(x-5)^2 - 4 = 0$

(12) $x^2 + 6x - 27 = 0$
$(x+3)^2 - 36 = 0$

(13) $x^2 - 4x + 3 = 0$
$(x-2)^2 - 1 = 0$

(14) $x^2 - 8x - 9 = 0$
$(x-4)^2 - 25 = 0$

Solve the following equations by completing the square.
These could not be solved by factorising.

(15) $x^2 - 2x - 2 = 0$
$(x-1)^2 - 3 = 0$

(16) $x^2 + 4x - 3 = 0$
$(x+2)^2 - 7 = 0$

(17) $x^2 - 6x + 13 = 0$
$(x-3)^2 + 4 = 0$

(18) $x^2 + 10x + 30 = 0$
$(x+5)^2 + 5 = 0$

Solve the following equations by completing the square.
Check your answer by factorising.

(19) $2x^2 - 12x + 16 = 0$
$2(x-3)^2 - 2 = 0$

(20) $3x^2 - 12x - 63 = 0$
$3(x-2)^2 - 75 = 0$

Quadratics - Completing the Square

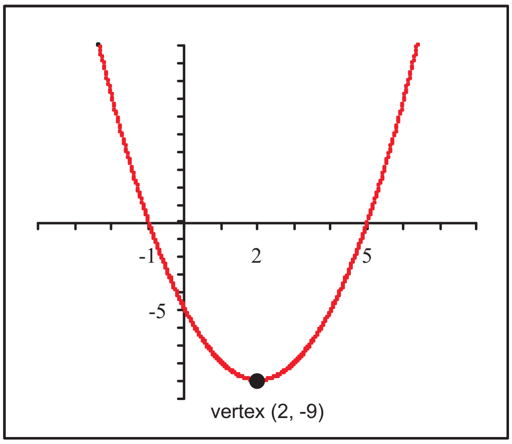
GRAPHICAL INFORMATION

The completed square form is useful as it gives you information about where the vertex (bottom or top) of a quadratic graph is.

- to find the x-value make the bracket zero
- the y-value is the number at the end.

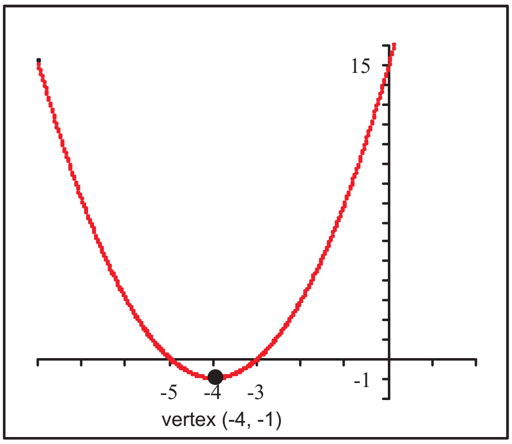
① $y = x^2 - 4x - 5$

$y = (x-2)^2 - 9$



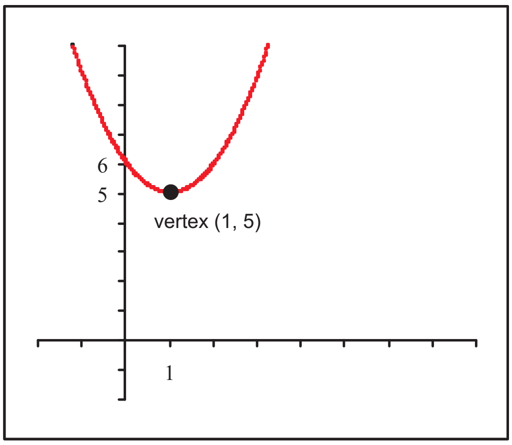
② $y = x^2 + 8x + 15$

$y = (x+4)^2 - 1$



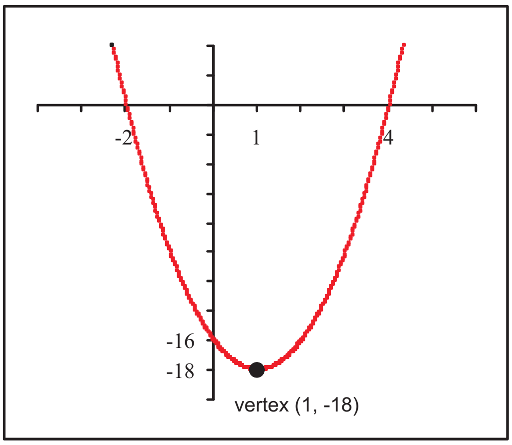
③ $y = x^2 - 2x + 6$

$y = (x-1)^2 + 5$



④ $y = 2x^2 - 4x - 16$

$y = 2(x-1)^2 - 18$



<u>Exercise C:</u> Graphical Information From the Completed Square Form

The **completed square** form is useful as it gives you information about where the **vertex** (**bottom** or **top**) of a **quadratic graph** is.

This exercise is designed to increase your awareness of how the **completed square form** relates to the **graph** of the **quadratic**.

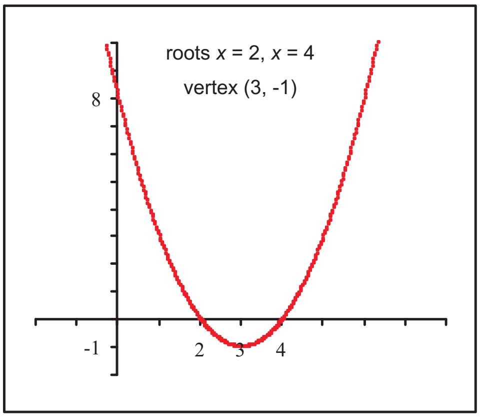
The **roots** from the **solution of the quadratic equation** (by **factorising/completing the square/quadratic formula**) are also marked as is the ***y*-intercept**.

These are graphs of ten of the quadratics from the first exercise.

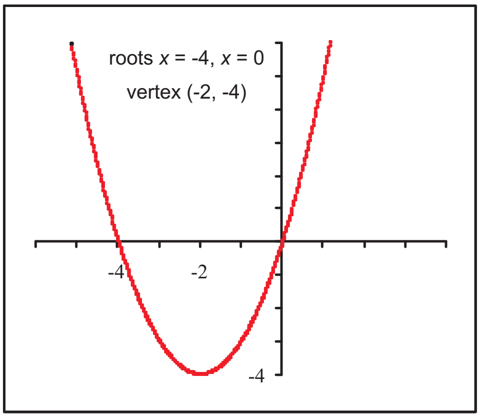
For each graph write below it:

the completed square form, the factorised form, the polynomial form.

(1)



(2)



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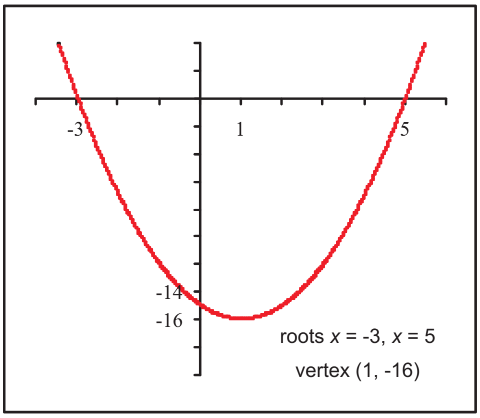

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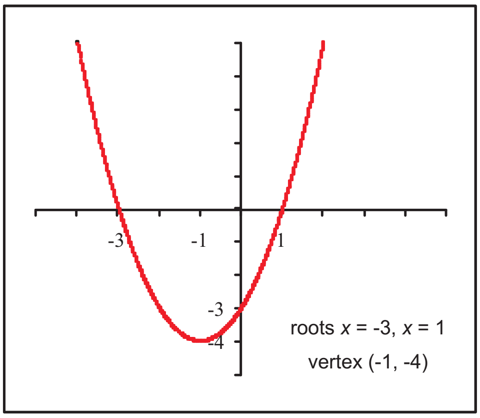
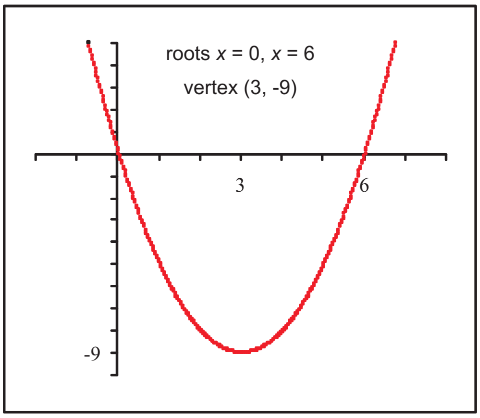


Quadratics - Completing the Square

Exercise C: Graphical Information From the Completed Square Form

(5)



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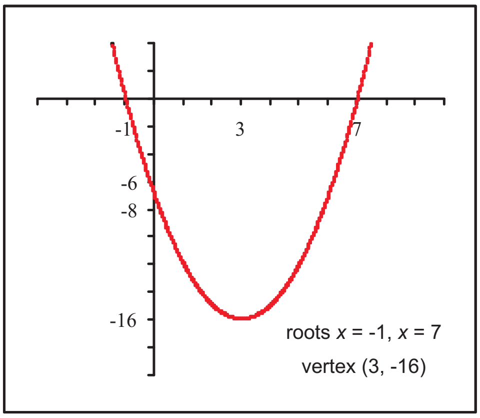
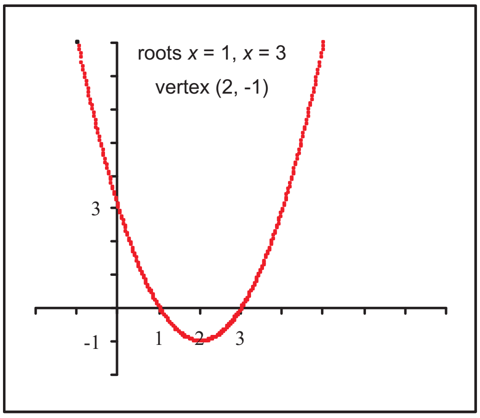
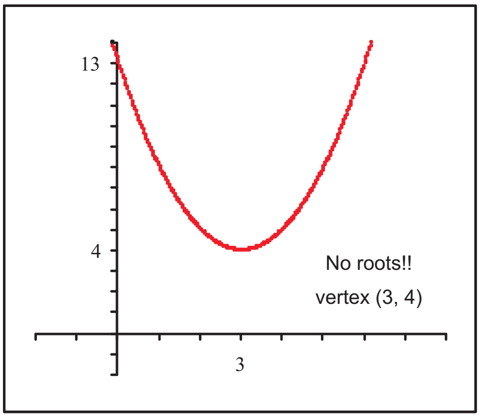


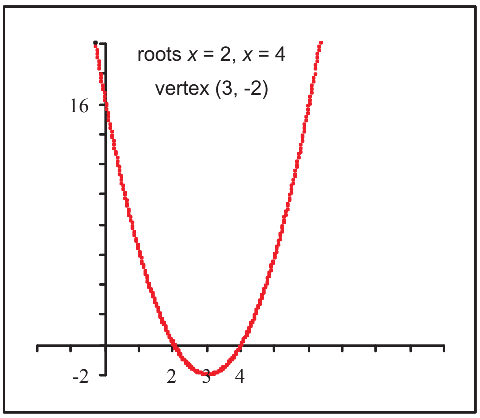
(7)



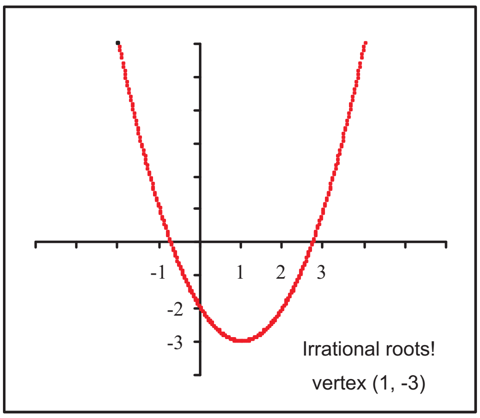
(8)



(9)

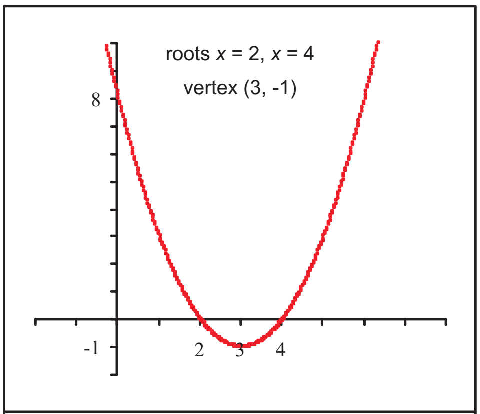


(10)



Exercise C: Graphical Information From the Completed Square Form

(1)



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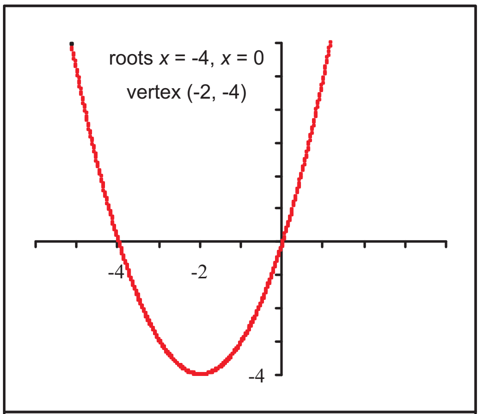

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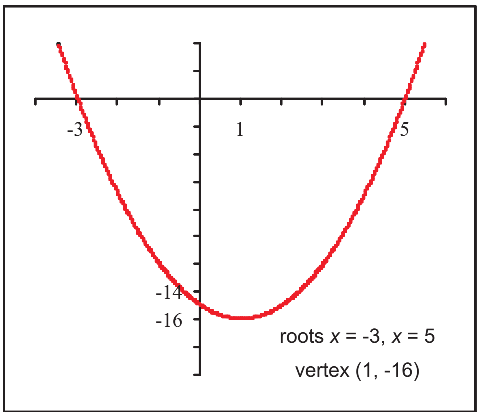

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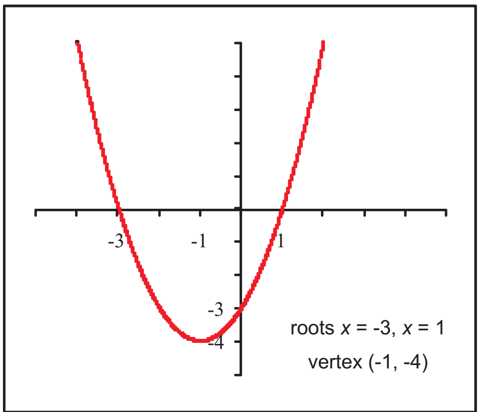
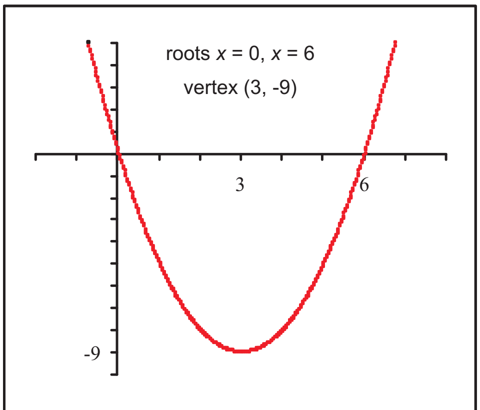


(5)



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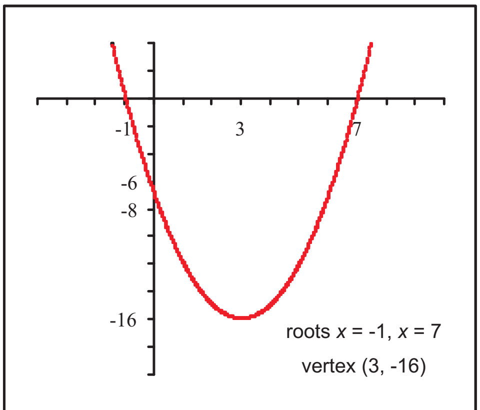
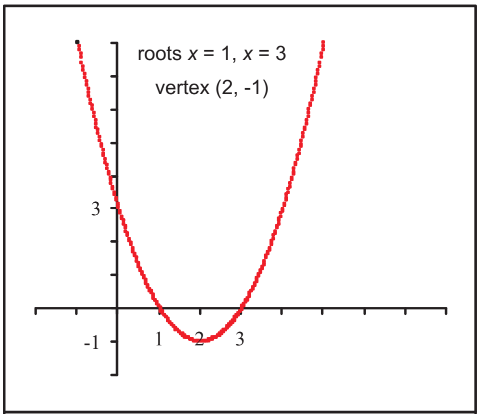


Exercise C: Graphical Information From the Completed Square Form

(7)



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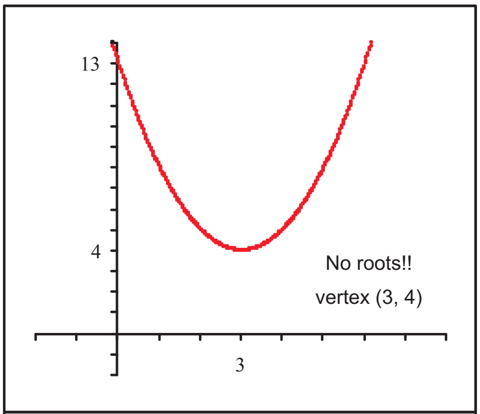
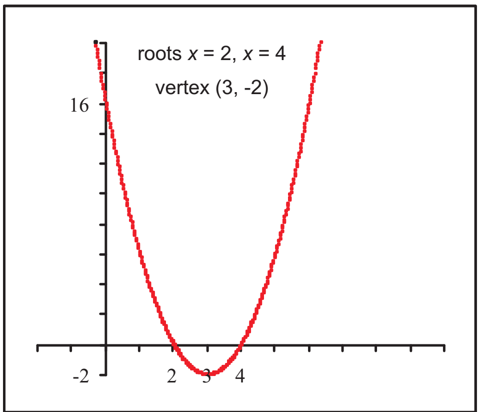


(9)



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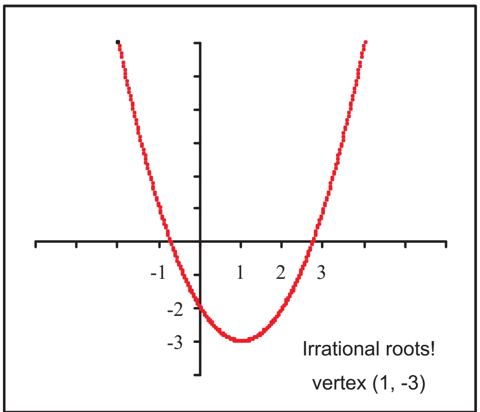


COMPLETING THE SQUARE — EXERCISE A

① x^2-6x+8
$(x-3)^2+8-9$
$(x-3)^2-1$

② x^2+2x-3
$(x+1)^2-3-1$
$(x+1)^2-4$

③ $x^2+12x+20$
$(x+6)^2+20-36$
$(x+6)^2-16$

④ x^2+4x
$(x+2)^2-4$

⑤ $x^2-2x-63$
$(x-1)^2-63-1$
$(x-1)^2-64$

⑥ $x^2-2x-15$
$(x-1)^2-15-1$
$(x-1)^2-16$

⑦ x^2-6x
$(x-3)^2-9$

⑧ $x^2+4x-21$
$(x+2)^2-21-4$
$(x+2)^2-25$

⑨ x^2-6x-7
$(x-3)^2-7-9$
$(x-3)^2-16$

⑩ x^2-2x
$(x-1)^2-1$

⑪ $x^2-10x+21$
$(x-5)^2+21-25$
$(x-5)^2-4$

⑫ $x^2+6x-27$
$(x+3)^2-27-9$
$(x+3)^2-36$

⑬ x^2-4x+3
$(x-2)^2+3-4$
$(x-2)^2-1$

⑭ x^2-8x-9
$(x-4)^2-9-16$
$(x-4)^2-25$

⑮ x^2-2x-2
$(x-1)^2-2-1$
$(x-1)^2-3$

⑯ x^2+4x-3
$(x+2)^2-3-4$
$(x+2)^2-7$

⑰ $x^2-6x+13$
$(x-3)^2+13-9$
$(x-3)^2+4$

⑱ $x^2+10x+30$
$(x+5)^2+30-25$
$(x+5)^2+5$

⑲ $2x^2-12x+16$
$2[x^2-6x]+16$
$2[(x-3)^2-9]+16$
$2(x-3)^2-18+16$
$2(x-3)^2-2$

⑳ $3x^2-12x-63$
$3[x^2-4x]-63$
$3[(x-2)^2-4]-63$
$3(x-2)^2-12-63$
$3(x-2)^2-75$

COMPLETING THE SQUARE — EXERCISE B

① $x^2-6x+8=0$
$(x-3)^2-1=0$
$(x-3)^2=1$
$x-3=\pm 1$
$x=3\pm 1$
$x=2, 4$

② $x^2+2x-3=0$
$(x+1)^2-4=0$
$(x+1)^2=4$
$x+1=\pm 2$
$x=-1\pm 2$
$x=-3, 1$

③ $x^2+12x+20=0$
$(x+6)^2-16=0$
$(x+6)^2=16$
$x+6=\pm 4$
$x=-6\pm 4$
$x=-10, -2$

④ $x^2+4x=0$
$(x+2)^2-4=0$
$(x+2)^2=4$
$x+2=\pm 2$
$x=-2\pm 2$
$x=-4, 0$

⑤ $x^2-2x-63=0$
$(x-1)^2-64=0$
$(x-1)^2=64$
$x-1=\pm 8$
$x=1\pm 8$
$x=-7, 9$

⑥ $x^2-2x-15=0$
$(x-1)^2-16=0$
$(x-1)^2=16$
$(x-1=\pm 4$
$x=1\pm 4$
$x=-3, 5$

⑦ $x^2-6x=0$
$(x-3)^2-9=0$
$(x-3)^2=9$
$x-3=\pm 3$
$x=3\pm 3$
$x=0, 6$

⑧ $x^2+4x-21=0$
$(x+2)^2-25=0$
$(x+2)^2=25$
$x+2=\pm 5$
$x=-2\pm 5$
$x=-7, 3$

COMPLETING THE SQUARE EXERCISE B

(9) $x^2-6x-7=0$
$(x-3)^2-16=0$
$(x-3)^2=16$
$x-3=\pm 4$
$x=3\pm 4$
$x=-1,7$

(10) $x^2-2x=0$
$(x-1)^2-1=0$
$(x-1)^2=1$
$x-1=\pm 1$
$x=1\pm 1$
$x=0,2$

(11) $x^2-10x+21=0$
$(x-5)^2-4=0$
$(x-5)^2=4$
$x-5=\pm 2$
$x=5\pm 2$
$x=3,7$

(12) $x^2+6x-27=0$
$(x+3)^2-36=0$
$(x+3)^2=36$
$x+3=\pm 6$
$x=-3\pm 6$
$x=-9,3$

(13) $x^2-4x+3=0$
$(x-2)^2-1=0$
$(x-2)^2=1$
$x-2=\pm 1$
$x=2\pm 1$
$x=1,3$

(14) $x^2-8x-9=0$
$(x-4)^2-25=0$
$(x-4)^2=25$
$x-4=\pm 5$
$x=4\pm 5$
$x=-1,9$

(15) $x^2-2x-2=0$
$(x-1)^2-3=0$
$(x-1)^2=3$
$x-1=\pm\sqrt{3}$
$x=1\pm\sqrt{3}$

(16) $x^2+4x-3=0$
$(x+2)^2-7=0$
$(x+2)^2=7$
$x+2=\pm\sqrt{7}$
$x=-2\pm\sqrt{7}$

(17) $x^2-6x+13=0$
$(x-3)^2+4=0$
$(x-3)^2=-4$

NO SOLUTIONS
CAN'T $\sqrt{-4}$

(18) $x^2+10x+30=0$
$(x+5)^2+5=0$
$(x+5)^2=-5$

NO SOLUTIONS
CAN'T $\sqrt{-5}$

(19) $2x^2-12x+16=0$
$2(x-3)^2-2=0$
$2(x-3)^2=2$
$(x-3)^2=1$
$x-3=\pm 1$
$x=3\pm 1$
$x=2,4$

(20) $3x^2-12x-63=0$
$3(x-2)^2-75=0$
$3(x-2)^2=75$
$(x-2)^2=25$
$x-2=\pm 5$
$x=2\pm 5$
$x=-3,7$

COMPLETING THE SQUARE EXERCISE C

(1) $(x-3)^2-1$
$(x-2)(x-4)$
x^2-6x+8

(2) $(x+2)^2-4$
$x(x+4)$
x^2+4x

(3) $(x-1)^2-16$
$(x+3)(x-5)$
$x^2-2x-15$

(4) $(x+1)^2-4$
$(x+3)(x-1)$
x^2+2x-3

(5) $(x-3)^2-9$
$x(x-6)$
x^2-6x

(6) $(x-3)^2-16$
$(x+1)(x-7)$
x^2-6x-7

(7) $(x-2)^2-1$
$(x-1)(x-3)$
x^2-4x+3

(8) $(x-3)^2+4$
NO FACTORS
$x^2-6x+13$

(9) $2(x-3)^2-2$
$2(x-2)(x-4)$
$2x^2-12x+16$

(10) $(x-1)^2-3$
IRRATIONAL FACTORS
x^2-2x-2

P. Leary

QUADRATICS

The Quadratic Formula

Objectives

- ★ learn the quadratic formula
- ★ solve quadratic equations using the quadratic formula
- ★ give answers in exact form or to a specified number of decimal places
- ★ be aware of different ways of solving quadratic equations
- ★ solve problems involving quadratic equations

REFERENCE IN OTHER RESOURCES

Head Start: Topic not covered.

Alpha Workbooks: Quadratic Formula - Pages 23-25

Quadratics - The Quadratic Formula

THE QUADRATIC FORMULA

Given a quadratic equation of the form:

$$ax^2 + bx + c = 0$$

The equation can be solved using the quadratic formula:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Always ensure that the quadratic equation is put in the form quadratic $= 0$ before solving!

There are various ways of solving quadratic equations:

- by factorising the quadratic (not all will factorise!)
- by completing the square
- using the quadratic formula

EXAMPLE

Use the quadratic equation to solve:

$$x^2 + 5x + 6 = 0$$

SOLUTION

$a = 1 \qquad b = 5 \qquad c = 6$

$$x = \frac{-5 \pm \sqrt{(5)^2 - (4 \times 1 \times 6)}}{2}$$

$$x = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{-5 \pm \sqrt{1}}{2}$$

$$x = \frac{-5 \pm 1}{2}$$

$$x = -\frac{6}{2} \quad \text{or} \quad x = -\frac{4}{2}$$

$$x = -3 \quad \text{or} \quad x = -2$$

* NOTE *

This illustrates the method but it would have been much easier to factorise!

$$(x+3)(x+2) = 0$$

$$x = -3 \quad x = -2$$

EXAMPLE

Use the quadratic formula to solve:

$$x^2 = 4x - 2$$

SOLUTION

Start by making quadratic $= 0$

$$x^2 - 4x + 2 = 0$$

$a = 1 \qquad b = -4 \qquad c = 2$

$$x = \frac{4 \pm \sqrt{(-4)^2 - (4 \times 1 \times 2)}}{2}$$

$$x = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$x = \frac{4 \pm \sqrt{8}}{2}$$

$$x = 3.41 \quad \text{or} \quad x = 0.59$$

* NOTE *

This quadratic equation would not factorise!

Completing the square would have been another option!

EXAMPLE

Solve $6x^2 + x - 12 = 0$

SOLUTION

$a = 6 \qquad b = 1 \qquad c = -12$

$$x = \frac{-1 \pm \sqrt{(1)^2 - (4 \times 6 \times -12)}}{2 \times 6}$$

$$x = \frac{-1 \pm \sqrt{1 - (-288)}}{12}$$

$$x = \frac{-1 \pm \sqrt{289}}{12}$$

$$x = \frac{-1 \pm 17}{12}$$

$$x = -\frac{18}{12} \quad \text{or} \quad x = \frac{16}{12}$$

$$x = -\frac{3}{2} \quad \text{or} \quad x = \frac{4}{3}$$

AFTER SIMPLIFYING

Quadratics - The Quadratic Formula

Exercise: Solving Equations using the Quadratic Formula

Given a **quadratic equation** of the form:

$$ax^2 + bx + c = 0$$

The **equation** can be solved in a number of ways:

- by **factorising** the quadratic {not all quadratics will factorise}
- by **completing the square**
- using the **quadratic formula**

The **quadratic formula** finds the solutions x of the equation by substitution:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Always ensure that the **quadratic equation** is put into the form **quadratic = 0** before using the formula to try and solve it; sometimes you have to manipulate the equation.

Use the quadratic formula to solve these equations (all will factorise):

(1) $x^2 + 8x + 15 = 0$

(2) $x^2 - 4x = 5$

(3) $2x^2 - 5x - 3 = 0$

(4) $x^2 + 2x + 1 = 0$

Use the quadratic formula to solve these equations, give the answer both in surd form and also to two decimal places (these will not factorise):

(5) $x^2 - 2x - 6 = 0$

(6) $5x^2 - 5x + 1 = 0$

(7) $x^2 = 1 - x$

(8) $x^2 = x + 5$

(9) The product of two consecutive positive numbers is 240.
Denoting the smaller number x then the problem is satisfied by the equation:

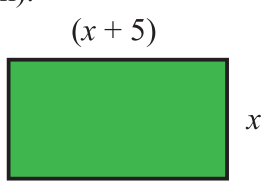
$$x(x+1) = 240$$

(a) show that this manipulates to the equation:

$$x^2 + x - 240 = 0$$

(b) solve the equation using the quadratic formula to find the two numbers

(10) A garden has a length which is 5 m more than its width (width = x m).
The area of the garden is 30 m^2.



(a) show that this problem satisfies the equation $x^2 + 5x - 30 = 0$

(b) solve the equation to find the width and length of the garden

QUADRATIC FORMULA EXERCISE

① $x^2 + 8x + 15 = 0$

$a = 1 \quad b = 8 \quad c = 15$

$$x = \frac{-8 \pm \sqrt{(8)^2 - (4 \times 1 \times 15)}}{2 \times 1}$$

$$x = \frac{-8 \pm \sqrt{64 - 60}}{2}$$

$$x = \frac{-8 \pm \sqrt{4}}{2}$$

$$x = \frac{-8 \pm 2}{2} = -5, -3$$

② $x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$a = 1 \quad b = -4 \quad c = -5$

$$x = \frac{4 \pm \sqrt{(-4)^2 - (4 \times 1 \times -5)}}{2 \times 1}$$

$$x = \frac{4 \pm \sqrt{16 - (-20)}}{2}$$

$$x = \frac{4 \pm \sqrt{36}}{2}$$

$$x = \frac{4 \pm 6}{2} = -1, 5$$

③ $2x^2 - 5x - 3 = 0$

$a = 2 \quad b = -5 \quad c = -3$

$$x = \frac{5 \pm \sqrt{(-5)^2 - (4 \times 2 \times -3)}}{2 \times 2}$$

$$x = \frac{5 \pm \sqrt{25 - (-24)}}{4}$$

$$x = \frac{5 \pm \sqrt{49}}{4}$$

$$x = \frac{5 \pm 7}{4} = -\frac{1}{2}, 3$$

④ $x^2 + 2x + 1 = 0$

$a = 1 \quad b = 2 \quad c = 1$

$$x = \frac{-2 \pm \sqrt{(2)^2 - (4 \times 1 \times 1)}}{2 \times 1}$$

$$x = \frac{-2 \pm \sqrt{4 - 4}}{2}$$

$$x = \frac{-2 \pm \sqrt{0}}{2}$$

$$x = \frac{-2}{2} = -1$$

QUADRATIC FORMULA EXERCISE

⑤ $x^2 - 2x - 6 = 0$

$a = 1 \quad b = -2 \quad c = -6$

$$x = \frac{2 \pm \sqrt{(-2)^2 - (4 \times 1 \times -6)}}{2 \times 1}$$

$$x = \frac{2 \pm \sqrt{4 - (-24)}}{2}$$

$$x = \frac{2 \pm \sqrt{28}}{2}$$

$x = -1.65, \; x = 3.65$

⑥ $5x^2 - 5x + 1 = 0$

$a = 5 \quad b = -5 \quad c = 1$

$$x = \frac{5 \pm \sqrt{(-5)^2 - (4 \times 5 \times 1)}}{2 \times 5}$$

$$x = \frac{5 \pm \sqrt{25 - 20}}{10}$$

$$x = \frac{5 \pm \sqrt{5}}{10}$$

$x = 0.28, \; x = 0.72$

⑦ $x^2 = 1 - x$

$x^2 + x - 1 = 0$

$a = 1 \quad b = 1 \quad c = -1$

$$x = \frac{-1 \pm \sqrt{(1)^2 - (4 \times 1 \times -1)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{1 - (-4)}}{2}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$x = -1.62, \; x = 0.62$

⑧ $x^2 = x + 5$

$x^2 - x - 5 = 0$

$a = 1 \quad b = -1 \quad c = -5$

$$x = \frac{1 \pm \sqrt{(-1)^2 - (4 \times 1 \times -5)}}{2 \times 1}$$

$$x = \frac{1 \pm \sqrt{1 - (-20)}}{2}$$

$$x = \frac{1 \pm \sqrt{21}}{2}$$

$x = -1.79, \; x = 2.79$

QUADRATIC FORMULA EXERCISE

(9) (a) 1ST NUMBER x
2ND NUMBER $x+1$

PRODUCT $x \times (x+1) = x(x+1)$

TOLD PRODUCT $= 240$

$\rightarrow \quad x(x+1) = 240$

EXPAND $x^2 + x = 240$

$x^2 + x - 240 = 0$

(b) $a = 1 \quad b = 1 \quad c = -240$

$$x = \frac{-1 \pm \sqrt{(1)^2 - (4 \times 1 \times -240)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{1 - (-960)}}{2}$$

$$x = \frac{-1 \pm \sqrt{961}}{2}$$

$$x = \frac{-1 \pm 31}{2} \rightarrow x = -16 \quad x = 15$$

IGNORE

NUMBERS ARE 15 AND 16 $\quad 15 \times 16 = 240$ ✓

(10) (a) $x(x+5) = 30$

$x^2 + 5x = 30$

$x^2 + 5x - 30 = 0$

$a = 1 \quad b = 5 \quad c = -30$

(b) $$x = \frac{-5 \pm \sqrt{(5)^2 - (4 \times 1 \times -30)}}{2 \times 1}$$

$$x = \frac{-5 \pm \sqrt{25 - (-120)}}{2}$$

$$x = \frac{-5 \pm \sqrt{145}}{2}$$

$x = -8.52 \quad x = 3.52$

IGNORE

GARDEN $3.52 \times 8.52 \approx 30$ ✓

P. Leary

EQUATIONS

Basic Equations and Equations Involving Fractions

Objectives

★ solve linear equations:

☆ including brackets and unknown on both sides

★ solve quadratic equations

★ solve equations involving fractions:

☆ where the denominators are numerical

★ solve equations involving algebraic fractions:

☆ where the unknown appears in the denominator

REFERENCE IN OTHER RESOURCES

Head Start: Algebraic Fractions - Page 13 (minimal coverage).

Alpha Workbooks: Linear Equations - Pages 13-17.

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Equations - Basic Equations and Equations Involving Fractions (Linear!)

LINEAR EQUATIONS REVIEW

- If an equation has x on both sides, collect all the x's on one side to solve
- If an equation has brackets, expand the brackets to remove them

EXAMPLE

Solve $3x+1 = x+7$

SOLUTION

$3x + 1 = x + 7$

$2x + 1 = +7$ SUBTRACT x FROM BOTH SIDES

$2x = 6$ SUBTRACT 1 FROM BOTH SIDES

$x = 3$ DIVIDE BOTH SIDES BY 2

EXAMPLE

Solve $5(2x-1) = 25$

SOLUTION

$5(2x-1) = 25$

$10x - 5 = 25$ EXPAND

$10x = 30$ $(+5)$ BOTH SIDES

$x = 3$ $(\div 10)$ BOTH SIDES

EXAMPLE

Solve $2(x+3) = 4x$

SOLUTION

$2(x+3) = 4x$

$2x + 6 = 4x$ EXPAND

$+6 = 2x$ $(-2x)$ BOTH SIDES

$3 = x$ $(\div 2)$ BOTH SIDES

EQUATIONS INVOLVING FRACTIONS I

Numbers on denominator.

Remove fractions by multiplying all terms by terms on denominators.

'MOVE BOTTOM' TO 'TOP' OF 'ALL OTHER TERMS'

EXAMPLE

Solve $\frac{3x+2}{5} = 4$

SOLUTION

$\frac{3x+2}{5} = 4$

$3x+2 = 5\times4$

$3x+2 = 20$

$3x = 18$

$x = 6$

EXAMPLE

Solve $\frac{3x+1}{2} + \frac{2x}{3} = 7$

SOLUTION

$3(3x+1) + 2\times2x = 2\times3\times7$

$9x + 3 + 4x = 42$

$13x + 3 = 42$

$13x = 39$

$x = 3$

Equations - Basic Equations and Equations Involving Fractions (Linear!)

EQUATIONS INVOLVING FRACTIONS II

Unknowns on denominator.

Remove fractions by multiplying all terms by terms on denominators.

'MOVE BOTTOM' TO 'TOP' OF 'ALL OTHER TERMS'

EXAMPLE

Solve $\frac{10}{x} = 2$

SOLUTION

$$\frac{10}{x} = 2$$
$$10 = x \times 2$$
$$10 = 2x$$
$$5 = x$$

EXAMPLE

Solve $\frac{16}{x+2} = 4$

SOLUTION

$$\frac{16}{x+2} = 4$$
$$16 = 4(x+2)$$
$$16 = 4x + 8$$
$$8 = 4x$$
$$2 = x$$

EXAMPLE

Solve $\frac{30}{x+1} = \frac{12}{x-2}$

SOLUTION

$$\frac{30}{x+1} = \frac{12}{x-2}$$
$$30(x-2) = 12(x+1)$$
$$30x - 60 = 12x + 12$$
$$18x - 60 = 12$$
$$18x = 72$$
$$x = \frac{72}{18} = 4$$

Equations - Basic Equations and Equations Involving Fractions (Linear!)

Exercise A: Basic Equation Review

Solve these **equations** to find the value of the unknown x.

(1) $2x+1=x+3$

(2) $5x-3=2x+9$

(3) $7x-10=2x+15$

(4) $4x+2=2x+3$

(5) $3(2x+3)=15$

(6) $4(7x-3)=44$

(7) $5(x-3)=x+1$

(8) $x+12=2(3-x)$

(9) $2x+3(x-4)=2x-3$

(10) $7x-6=5x-2(x-1)$

Exercise B: Equations involving Fractions (Unknown Numerator Only)

Solve these **equations** to find the value of the unknown x.

There are **fractions present**, start by removing these!

(1) $\frac{x-1}{3}=1$

(2) $\frac{5x-1}{7}=2$

(3) $\frac{2x+1}{2}=4$

(4) $\frac{3x+8}{5}+\frac{x}{2}=6$

(5) $\frac{5x+7}{2}-\frac{x+5}{3}=4$

(6) $\frac{5x-3}{4}+\frac{x+5}{3}=12$

(7) $\frac{2x+3}{3}-\frac{2x-2}{5}=3$

(8) $\frac{7x+4}{2}+\frac{x+3}{5}=10$

(9) $\frac{x+1}{2}=\frac{3x-1}{5}$

(10) $\frac{2x+3}{3}=\frac{4x+3}{5}$

Equations - Basic Equations and Equations Involving Fractions (Linear!)

Exercise C: Equations involving Fractions (Unknown on Denominator)

Solve these **equations** to find the value of the unknown x.

There are **fractions present** with the **unknown** in the **denominator**, start by removing these!

(1) $\frac{20}{x} = 5$

(2) $\frac{15}{x} = 5$

(3) $\frac{32}{x} = 4$

(4) $\frac{25}{x+3} = 5$

(5) $\frac{24}{5x+1} = 4$

(6) $\frac{14}{3x-7} = 7$

(7) $\frac{10}{x-1} = \frac{5}{2x-5}$

(8) $\frac{12}{x-2} = \frac{8}{x-3}$

(9) $\frac{4}{x+2} - \frac{3}{x} = \frac{1}{x-1}$

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Equations - Equations and Equations Involving Fractions (Quadratic!)

EQUATIONS INVOLVING QUADRATICS

Some equations, possibly involving fractions, will reduce to a quadratic equation.

Ensure you have quadratic = 0, then solve (attempting to factorise first).

EXAMPLE

Solve $x(x+4) = 2x+3$

SOLUTION

$$x(x+4) = 2x+3$$
$$x^2+4x = 2x+3$$
$$x^2+4x-2x-3 = 0$$
$$x^2+2x-3 = 0$$
$$(x+3)(x-1) = 0$$
$$x=-3 \quad x=1$$

EXAMPLE

Solve $x+1 = \frac{6}{x}$

SOLUTION

$$x+1 = \frac{6}{x} \rightarrow x(x+1) = 6$$
$$x^2+x = 6$$
$$x^2+x-6 = 0$$
$$(x+3)(x-2) = 0$$
$$x=-3 \quad x=2$$

EQUATIONS INVOLVING QUADRATICS

Some questions involving algebraic fractions reduce to quadratics.

EXAMPLE

Solve $\frac{2}{x} = \frac{x}{x+4}$

SOLUTION

$$\frac{2}{x} = \frac{x}{x+4}$$

MOVE BOTTOMS TO TOP OF EVERYWHERE ELSE!

$$2(x+4) = x \times x$$
$$2x+8 = x^2$$
$$0 = x^2-2x-8$$
$$0 = (x-4)(x+2)$$
$$x=4 \quad x=-2$$

EQUATIONS INVOLVING QUADRATICS

Some equations involving algebraic fractions reduce to quadratics.

EXAMPLE

Solve $\frac{3}{x+5} + \frac{2}{x-1} = 1$

SOLUTION

$$\frac{3}{x+5} + \frac{2}{x-1} = 1$$

MOVE BOTTOMS TO TOP OF EVERYWHERE ELSE!

$$3(x-1) + 2(x+5) = 1(x+5)(x-1)$$
$$3x-3+2x+10 = x^2+4x-5$$
$$5x+7 = x^2+4x-5$$
$$0 = x^2+4x-5-5x-7$$
$$0 = x^2-x-12$$
$$0 = (x+3)(x-4)$$
$$x=-3 \quad x=4$$

Equations - Equations and Equations Involving Fractions (Quadratic!)

<u>Exercise D:</u> Equations that Reduce to Quadratics (Including Algebraic Fractions)

Solve these **equations** to find the value of the unknown x.

All reduce to **quadratics**, if there are **fractions present** start by removing these!

(1) $x(x+2) = 2x+9$

(2) $x(x-8) - 2(x-12) = 0$

(3) $x + 2 = \dfrac{15}{x}$

(4) $2x + \dfrac{3}{x} = 7$

(5) $\dfrac{4}{x+3} = \dfrac{x+1}{2x}$

(6) $\dfrac{3x+2}{5x} = \dfrac{4}{x+3}$

(7) $\dfrac{x}{4} = \dfrac{2x}{x+7}$

(8) $\dfrac{4}{x} - \dfrac{2}{x+3} = 1$

(9) $\dfrac{6}{x+4} + \dfrac{4}{x-2} = 2$

(10) $\dfrac{5}{x-2} - \dfrac{2}{x} = 2$

(11) $\dfrac{7}{x+3} + \dfrac{2}{3x+1} = 1$

(12) $\dfrac{5}{x+1} + \dfrac{3}{2x} = 4$

EQUATIONS

EXERCISE A

① $2x+1 = x+3$

$x+1 = 3$

$x = 2$

② $5x-3 = 2x+9$

$3x-3 = 9$

$3x = 12$

$x = 4$

③ $7x-10 = 2x+15$

$5x-10 = 15$

$5x = 25$

$x = 5$

④ $4x+2 = 2x+3$

$2x+2 = 3$

$2x = 1$

$x = 1/2$

⑤ $3(2x+3) = 15$

$6x+9 = 15$

$6x = 6$

$x = 1$

⑥ $4(7x-3) = 44$

$28x-12 = 44$

$28x = 56$

$x = 2$

⑦ $5(x-3) = x+1$

$5x-15 = x+1$

$4x-15 = 1$

$4x = 16$

$x = 4$

⑧ $x+12 = 2(3-x)$

$x+12 = 6-2x$

$3x+12 = 6$

$3x = -6$

$x = -2$

⑨ $2x+3(x-4) = 2x-3$

$2x+3x-12 = 2x-3$

$5x-12 = 2x-3$

$3x-12 = -3$

$3x = 9$

$x = 3$

⑩ $7x-6 = 5x-2(x-1)$

$7x-6 = 5x-2x+2$

$7x-6 = 3x+2$

$4x-6 = 2$

$4x = 8$

$x = 2$

EQUATIONS EXERCISE B

① $\frac{x-1}{3} = 1$

$x-1 = 3\times 1$

$x-1 = 3$

$x = 4$

② $\frac{5x-1}{7} = 2$

$5x-1 = 7\times 2$

$5x-1 = 14$

$5x = 15$

$x = 3$

③ $\frac{2x+1}{2} = 4$

$2x+1 = 2\times 4$

$2x+1 = 8$

$2x = 7$

$x = 7/2$

④ $\frac{3x+8}{5} + \frac{x}{2} = 6$

$2(3x+8) + 5x = 5\times 2\times 6$

$6x + 16 + 5x = 60$

$11x + 16 = 60$

$11x = 44$

$x = 4$

⑤ $\frac{5x+7}{2} - \frac{x+5}{3} = 4$

$3(5x+7) - 2(x+5) = 2\times 3\times 4$

$15x + 21 - 2x - 10 = 24$

$13x + 11 = 24$

$13x = 13$

$x = 1$

⑥ $\frac{5x-3}{4} + \frac{x+5}{3} = 12$

$3(5x-3) + 4(x+5) = 4\times 3\times 12$

$15x - 9 + 4x + 20 = 144$

$19x + 11 = 144$

$19x = 133$

$x = 7$

⑦ $\frac{2x+3}{3} - \frac{2x-2}{5} = 3$

$5(2x+3) - 3(2x-2) = 3\times 5\times 3$

$10x + 15 - 6x + 6 = 45$

$4x + 21 = 45$

$4x = 24$

$x = 6$

⑧ $\frac{7x+4}{2} + \frac{x+3}{5} = 10$

$5(7x+4) + 2(x+3) = 2\times 5\times 10$

$35x + 20 + 2x + 6 = 100$

$37x + 26 = 100$

$37x = 74$

$x = 2$

⑨ $\frac{x+1}{2} = \frac{3x-1}{5}$

$5(x+1) = 2(3x-1) \rightarrow x = 7$

⑩ $\frac{2x+3}{3} = \frac{4x+3}{5}$

$5(2x+3) = 3(4x+3)$

$\rightarrow x = 3$

EQUATIONS EXERCISE C

① $\frac{20}{x} = 5$

$20 = 5x$

$4 = x$

② $\frac{15}{x} = 5$

$15 = 5x$

$3 = x$

③ $\frac{32}{x} = 4$

$32 = 4x$

$8 = x$

④ $\frac{25}{x+3} = 5$

$25 = 5(x+3)$

$25 = 5x+15$

$10 = 5x$

$2 = x$

⑤ $\frac{24}{5x+1} = 4$

$24 = 4(5x+1)$

$24 = 20x+4$

$20 = 20x$

$1 = x$

⑥ $\frac{14}{3x-7} = 7$

$14 = 7(3x-7)$

$14 = 21x-49$

$63 = 21x$

$3 = x$

⑦ $\frac{10}{x-1} = \frac{5}{2x-5}$

$10(2x-5) = 5(x-1)$

$20x-50 = 5x-5$

$15x = 45$

$x = 3$

⑧ $\frac{12}{x-2} = \frac{8}{x-3}$

$12(x-3) = 8(x-2)$

$12x-36 = 8x-16$

$4x-36 = -16$

$4x = 20$

$x = 5$

⑨ $\frac{4}{x+2} - \frac{3}{x} = \frac{1}{x-1}$

$4(x-1)x - 3(x+2)(x-1) = (x+2)x$

$4(x^2-x) - 3(x^2+x-2) = x^2+2x$

$4x^2-4x - 3x^2-3x+6 = x^2+2x$

$x^2-7x+6 = x^2+2x$

$-7x+6 = 2x$

$6 = 9x$

$\frac{6}{9} = x$

$x = \frac{2}{3}$

EQUATIONS

EXERCISE D

① $x(x+2) = 2x+9$

$x^2+2x = 2x+9$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3 \quad x = 3$

② $x(x-8) - 2(x-12) = 0$

$x^2 - 8x - 2x + 24 = 0$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x = 4 \quad x = 6$

③ $x + 2 = \frac{15}{x}$

$x(x+2) = 15$

$x^2+2x = 15$

$x^2+2x-15 = 0$

$(x-3)(x+5) = 0$

$x = 3 \quad x = -5$

④ $2x + \frac{3}{x} = 7$

$2x^2 + 3 = 7x$

$2x^2 - 7x + 3 = 0$

$(2x-1)(x-3) = 0$

$x = \frac{1}{2} \quad x = 3$

⑤ $\frac{4}{x+3} = \frac{x+1}{2x}$

$4 \times 2x = (x+3)(x+1)$

$8x = x^2+4x+3$

$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$x = 1 \quad x = 3$

⑥ $\frac{3x+2}{5x} = \frac{4}{x+3}$

$(3x+2)(x+3) = 4 \times 5x$

$3x^2 + 11x + 6 = 20x$

$3x^2 - 9x + 6 = 0$

$3(x^2 - 3x + 2) = 0$

$3(x-1)(x-2) = 0$

$x = 1 \quad x = 2$

⑦ $\frac{x}{4} = \frac{2x}{x+7}$

$x(x+7) = 2x \times 4$

$x^2 + 7x = 8x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, \ x = 1$

EQUATIONS EXERCISE D

(8) $\frac{4}{x} - \frac{2}{x+3} = 1$

$4(x+3) - 2x = 1x(x+3)$

$4x+12 - 2x = x^2+3x$

$2x+12 = x^2+3x$

$0 = x^2+x-12$

$0 = (x+4)(x-3)$

$x=-4 \quad x=3$

(9) $\frac{6}{x+4} + \frac{4}{x-2} = 2$

$6(x-2) + 4(x+4) = 2(x+4)(x-2)$

$6x-12 + 4x+16 = 2x^2 +4x -16$

$10x + 4 = 2x^2+4x-16$

$0 = 2x^2-6x-20$

$0 = 2(x^2-3x-10)$

$0 = 2(x-5)(x+2)$

$x=5 \quad x=-2$

(10) $\frac{5}{x-2} - \frac{2}{x} = 2$

$5x - 2(x-2) = 2x(x-2)$

$5x - 2x + 4 = 2x^2 - 4x$

$3x + 4 = 2x^2-4x$

$0 = 2x^2-7x-4$

$0 = (2x+1)(x-4)$

$x=-1/2 \quad x=4$

(11) $\frac{7}{x+3} + \frac{2}{3x+1} = 1$

$7(3x+1) + 2(x+3) = 1(x+3)(3x+1)$

$21x+7 +2x+6 = 3x^2+10x+3$

$23x+13 = 3x^2+10x+3$

$0 = 3x^2-13x-10$

$0 = (x-5)(3x+2)$

$x=5, \; x=-2/3$

(12) $\frac{5}{x+1} + \frac{3}{2x} = 4$

$5\times 2x + 3(x+1) = 4\times 2x(x+1)$

$10x + 3x+3 = 8x^2+8x$

$13x+3 = 8x^2+8x$

$0 = 8x^2-5x-3$

$(x-1)(8x+3)=0$

$x=1 \quad x=-3/8$

FRACTIONS

Numerical and Algebraic Fractions

Objectives

★ be confident with adding, subtracting, multiplying and dividing fractions

★ know that dividing by a fraction is the same as multiplying by its reciprocal

★ substitute fractions and negative numbers into formulae

★ simplify a single algebraic fraction by cancelling common factors

- ☆ including when the numerator/denominator need to be factorised first

★ add or subtract algebraic fractions fully simplifying the result

★ multiply or divide algebraic fractions fully simplifying the result

REFERENCE IN OTHER RESOURCES

Head Start: Fractions - Pages 3-5; Algebraic Fractions - Pages 12-14.

Alpha Workbooks: Fractions - Pages 3-5; Substitution - Pages 10-12.

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Fractions - Numerical Review

MIXED NUMBERS AND IMPROPER FRACTIONS

MIXED NUMBER	IMPROPER FRACTION (top-heavy, vulgar)
$2\frac{1}{2}$	$\frac{5}{2}$
$1\frac{4}{5}$	$\frac{9}{5}$
$6\frac{2}{3}$	$\frac{20}{3}$
$3\frac{7}{10}$	$\frac{37}{10}$

Know how to convert between mixed numbers and improper fractions.

Know that for a lot of algebraic work, the improper fraction is the most useful form.

EXAMPLE

Write $3\frac{7}{10}$ as a top heavy fraction

SOLUTION

$3\frac{7}{10} = \frac{?}{10}$ ← same denominator

$? = \underset{\text{BIG}}{3} \times \underset{\text{BOTTOM}}{10} + 7$

$3\frac{7}{10} = \frac{37}{10}$

EXAMPLE

Write $\frac{20}{3}$ as a mixed number

SOLUTION

how many 3's in 20 → 6
remainder is 2

$\frac{20}{3} = \underset{\text{full ones}}{6}\frac{2}{3}$ ← remainder

ADDING AND SUBTRACTING FRACTIONS

When adding or subtracting fractions, you must have a common denominator.

Do the 'whole' and 'fraction parts' seperately when dealing with mixed numbers.

EXAMPLE → SOLUTION

$\frac{4}{5} + \frac{1}{10}$

LCM 5, 10 = 10

$\frac{4^{\times 2}}{5_{\times 2}} + \frac{1}{10} = \frac{8}{10} + \frac{1}{10} = \frac{9}{10}$

EXAMPLE → SOLUTION

$\frac{2}{3} + \frac{1}{2}$

LCM 3, 2 = 6

$\frac{2^{\times 2}}{3_{\times 2}} + \frac{1^{\times 3}}{2_{\times 3}} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$ or $1\frac{1}{6}$

EXAMPLE → SOLUTION

$\frac{2}{3} - \frac{1}{4}$

LCM 3, 4 = 12

$\frac{2^{\times 4}}{3_{\times 4}} - \frac{1^{\times 3}}{4_{\times 3}} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$

EXAMPLE → SOLUTION

$1\frac{3}{4} + 2\frac{4}{5}$

LCM 4, 5 = 20

$1\frac{3}{4} + 2\frac{4}{5} = \underset{\text{wholes done}}{3}\frac{15}{20} + \frac{16}{20} = 3\frac{31}{20}$

but $\frac{31}{20} = 1\frac{11}{20}$ ∴ ANSWER $4\frac{11}{20}$

MULTIPLYING FRACTIONS

- write any mixed numbers as improper fractions
- write integers as themselves over one
- multiply by doing $\frac{\text{top} \times \text{top}}{\text{bottom} \times \text{bottom}}$

EXAMPLE → SOLUTION

$\frac{2}{3} \times \frac{3}{4}$

$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$

EXAMPLE → SOLUTION

$\frac{4}{5} \times \frac{1}{2}$

$\frac{4}{5} \times \frac{1}{2} = \frac{4 \times 1}{5 \times 2} = \frac{4}{10} = \frac{2}{5}$

EXAMPLE → SOLUTION

$5 \times \frac{2}{15}$

$5 \times \frac{2}{15} = \frac{5}{1} \times \frac{2}{15} = \frac{5 \times 2}{1 \times 15} = \frac{10}{15}$

$= \frac{2}{3}$

EXAMPLE → SOLUTION

$1\frac{2}{3} \times 4\frac{1}{2}$

$1\frac{2}{3} \times 4\frac{1}{2} = \frac{5}{3} \times \frac{9}{2}$

$= \frac{5 \times 9}{3 \times 2} = \frac{45}{6} = \frac{15}{2} = 7\frac{1}{2}$

Note:

With more experience, some cancelling can be done mid-calculation to avoid large numbers!

DIVIDING FRACTIONS

- write any mixed numbers as improper fractions
- write integers as themselves over one
- to divide → flip the second fraction upside down
  then multiply the fractions

EXAMPLE → SOLUTION

$\frac{1}{2} \div \frac{3}{5}$

$\frac{1}{2} \div \frac{3}{5}$

$\frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$

EXAMPLE → SOLUTION

$\frac{3}{4} \div 2$

$\frac{3}{4} \div \frac{2}{1}$

$\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$

EXAMPLE → SOLUTION

$2\frac{3}{5} \div 1\frac{3}{4}$

$\frac{13}{5} \div \frac{7}{4}$

$\frac{13}{5} \times \frac{4}{7} = \frac{13 \times 4}{5 \times 7} = \frac{52}{35} = 1\frac{17}{35}$

DIVIDING BY A FRACTION IS THE SAME AS MULTIPLYING BY ITS RECIPROCAL!

Fractions - Numerical Review

Exercise A: Mixed Fraction Arithmetic (Review)

As part of the course you will need to be very confident in working with **fractions** and **negative numbers**.

The work here focuses on fractions.

Fractions more than a whole can be written either as **mixed numbers** or **improper (top heavy) fractions**.

For each fraction below, give the equivalent fraction in the form other than that presented.

(1) $1\frac{2}{3}$ (2) $6\frac{1}{4}$ (3) $2\frac{4}{5}$

(4) $\frac{27}{10}$ (5) $\frac{31}{8}$ (6) $\frac{15}{2}$

When **adding** or **subtracting fractions** you need to ensure that you have a **common denominator**.

Complete these fraction addition and subtraction questions, simplifying if possible.

(7) $\frac{1}{2}+\frac{2}{5}$ (8) $\frac{9}{10}-\frac{2}{3}$ (9) $\frac{3}{4}+\frac{5}{7}$

(10) $1\frac{1}{3}+4\frac{1}{4}$ (11) $7\frac{5}{6}-3\frac{1}{8}$ (12) $5\frac{2}{5}-2\frac{2}{3}$

When **multiplying fractions**:

- write any **mixed numbers** as **improper fractions**
- write **integers** as **themselves over one**
- **multiply** the **top numbers together** and the **bottom numbers together**

Complete these fraction multiplication questions (simplify the result if possible).

(13) $\frac{2}{3}\times\frac{3}{7}$ (14) $\frac{4}{9}\times\frac{7}{10}$ (15) $\frac{1}{2}\times\frac{3}{5}$

(16) $\frac{3}{4}\times1\frac{1}{3}$ (17) $5\times3\frac{1}{4}$ (18) $2\frac{3}{5}\times4\frac{1}{9}$

When **dividing fractions**:

- write any **mixed numbers** as **improper fractions**
- write **integers** as **themselves over one**
- **flip** the **second fraction upside down** then **multiply the fractions**

Complete these fraction division questions (simplify the result if possible).

(19) $\frac{3}{10}\div\frac{1}{2}$ (20) $\frac{7}{12}\div\frac{3}{4}$ (21) $\frac{1}{3}\div2=\frac{1}{3}\div\frac{2}{1}$

(22) $2\frac{1}{2}\div3\frac{2}{3}$ (23) $\dfrac{\frac{3}{4}}{2}$ (24) $\dfrac{6}{\frac{2}{3}}$

Fractions - Numerical Review

Exercise A: Mixed Fraction Arithmetic (Review)

As part of the course you will need to be very confident in working with **fractions** and **negative numbers**.

The work here focuses on fractions.

Complete these statements about division involving fractions:

(25) dividing by $\frac{1}{2}$ is the same as multiplying by ……

(26) dividing by $\frac{1}{5}$ is the same as multiplying by ……

(27) dividing by $\frac{2}{3}$ is the same as multiplying by ……

(28) dividing by 2 is the same as multiplying by ……

Find **half** of each of these **fractions**:

(29) $\frac{4}{5}$ (30) $\frac{3}{7}$ (31) $2\frac{3}{4}$

Work out these calculations involving **fractions** to **powers**.

(32) $\left(\frac{1}{2}\right)^2$ (33) $\left(\frac{3}{4}\right)^2$ (34) $\left(\frac{2}{5}\right)^3$

(35) $\left(\frac{3}{10}\right)^4$ (36) $\left(1\frac{2}{3}\right)^3$ (37) $\left(2\frac{1}{2}\right)^2$

Fractions - Numerical Review

Exercise B: Substitution into Formulae

For each **linear function** given, find the ***y*-coordinate** given the ***x*-coordinate**.

(1) Find the y-value on the line graph defined by

$$y = 2x - 3$$

Given the following x-values:

(a) 2 (b) –4 (c) $\frac{1}{2}$ (d) $-\frac{3}{8}$

(2) Find the y-value on the line graph defined by

$$y = 2 - x$$

Given the following x-values:

(a) 3 (b) –5 (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

(3) Find the y-value on the line graph defined by

$$y = 7 - \tfrac{3}{2}x$$

Given the following x-values:

(a) 4 (b) –3 (c) $\frac{3}{2}$ (d) $-\frac{1}{4}$

For each **quadratic function** given, find the ***y*-coordinate** given the ***x*-coordinate**.

(4) Find the y-value on the quadratic curve defined by

$$y = x^2 + 2x$$

Given the following x-values:

(a) 4 (b) –2 (c) $\frac{1}{2}$ (d) $-\frac{1}{4}$

(5) Find the y-value on the quadratic curve defined by

$$y = x^2 - 3x + 5$$

Given the following x-values:

(a) 5 (b) –3 (c) $\frac{2}{3}$ (d) $-\frac{1}{2}$

(6) Find the y-value on the quadratic curve defined by

$$y = 4x^2 - x - 7$$

Given the following x-values:

(a) 2 (b) –1 (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

Fractions - Numerical Review

Exercise B: Substitution into Formulae

For each **formula** given, **substitute** in the **given values**.

(7) Find the y-value on the cubic curve defined by

$$y = 4x^3$$

Given the following x-values:

(a) 5 (b) –3 (c) $\frac{2}{5}$ (d) $-\frac{1}{4}$

(8) Find the y-value on the cubic curve defined by

$$y = x^3 + 5x^2 - 2x$$

Given the following x-values:

(a) 10 (b) –2 (c) $\frac{3}{4}$ (d) $-\frac{1}{2}$

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Fractions - Algebraic Fractions

SIMPLIFYING ALGEBRAIC FRACTIONS

Ideas from numerical fraction work also apply to algebraic fractions.

When simplifying algebraic fractions, cancel common factors on the numerator and denominator (these common factors must be multiplied by other factors!).

EXAMPLE → SOLUTION

Simplify $\frac{6p^4}{10p^2}$

$\frac{6\cancel{p}\times\cancel{p}\times p\times p}{10\cancel{p}\times\cancel{p}} = \frac{3p^2}{5}$

cancel numbers as well

can also use laws of indices on this question type.

EXAMPLE → SOLUTION

Simplify $\frac{2x(x+1)^2}{x^2(x+1)}$

$\frac{2\cancel{x}\cancel{(x+1)}(x+1)}{\cancel{x}x\cancel{(x+1)}} = \frac{2(x+1)}{x}$

EXAMPLE → SOLUTION

Simplify $\frac{4x+8}{6x+12}$

$\frac{4x+8}{6x+12} = \frac{4\cancel{(x+2)}}{6\cancel{(x+2)}} = \frac{4}{6} = \frac{2}{3}$

EXAMPLE → SOLUTION

Simplify $\frac{x-3}{x^2-9}$

$\frac{x-3}{x^2-9} = \frac{\cancel{x-3}}{(x+3)\cancel{(x-3)}} = \frac{1}{x+3}$

Note:

In more difficult questions (like the last two) you will have to factorise the numerator/denominator before factors are obvious.

LOWEST COMMON MULTIPLES

The lowest common multiple (LCM) should be used to obtain a common denominator when adding/subtracting fractions.

TERM ONE	TERM TWO	LOWEST COMMON MULTIPLE (LCM)
x	$2x$	$2x$ *
a	b	ab
$2p$	q	$2pq$
$4x$	$2y$	$4xy$ *
$(x+1)$	x	$x(x+1)$
$(x-5)$	$(x+3)$	$(x-5)(x+3)$

In all cases a common multiple can be found by multiplying the terms together.

This will not always be the LCM as * results illustrate (here there was an additional common factor between the terms).

ADDING/SUBTRACTING ALGEBRAIC FRACTIONS

Ideas from numerical work also apply to algebraic fractions.

Find a common denominator before adding or subtracting fractions.
(Look for the LCM of all denominators present).

EXAMPLE → SOLUTION LCM = 12

$\frac{2x}{3} + \frac{x}{4}$

$\frac{2x}{3} + \frac{x}{4} = \frac{8x}{12} + \frac{3x}{12} = \frac{11x}{12}$

EXAMPLE → SOLUTION LCM = 6

$\frac{x+1}{3} + \frac{2x-1}{2}$

$\frac{x+1}{3} + \frac{2x-1}{2} = \frac{2(x+1)}{6} + \frac{3(2x-1)}{6}$

$= \frac{2(x+1)+3(2x-1)}{6}$

$= \frac{2x+2+6x-3}{6}$

$= \frac{8x-1}{6}$

EXAMPLE → SOLUTION LCM = 12

$\frac{2x-5}{4} - \frac{x-1}{3}$

$\frac{2x-5}{4} - \frac{x-1}{3} = \frac{3(2x-5)}{12} - \frac{4(x-1)}{12}$

$= \frac{3(2x-5)-4(x-1)}{12}$

$= \frac{6x-15-4x+4}{12}$

$= \frac{2x-11}{12}$

ADDING/SUBTRACTING ALGEBRAIC FRACTIONS

EXAMPLE → SOLUTION LCM = 2x

$\frac{3}{x} + \frac{1}{2x}$

$\frac{3}{x} + \frac{1}{2x} = \frac{6}{2x} + \frac{1}{2x} = \frac{7}{2x}$

EXAMPLE → SOLUTION LCM $(x+1)(x-4)$

$\frac{4}{x+1} + \frac{2}{x-4}$

$\frac{4}{x+1} + \frac{2}{x-4}$

$= \frac{4(x-4)}{(x+1)(x-4)} + \frac{2(x+1)}{(x+1)(x-4)}$

$= \frac{4(x-4)+2(x+1)}{(x+1)(x-4)}$

$= \frac{4x-16+2x+2}{(x+1)(x-4)}$

$= \frac{2x-14}{(x+1)(x-4)}$

$= \frac{2(x-7)}{(x+1)(x-4)}$

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Fractions - Algebraic Fractions

MULTIPLYING ALGEBRAIC FRACTIONS

Ideas from numerical work apply to algebraic fractions.

When multiplying do: $\dfrac{\text{top} \times \text{top}}{\text{bottom} \times \text{bottom}}$

Simplify the result if possible.

EXAMPLE → SOLUTION

$\dfrac{x-2}{3x} \times \dfrac{2x}{(x-2)^2}$

$$\frac{x-2}{3x} \times \frac{2x}{(x-2)^2}$$

$$= \frac{2x(x-2)}{3x(x-2)^2}$$

$$= \frac{2\cancel{x}\cancel{(x-2)}}{3\cancel{x}\cancel{(x-2)}(x-2)}$$

$$= \frac{2}{3(x-2)}$$

EXAMPLE → SOLUTION

$\dfrac{x+5}{2} \times \dfrac{8}{x^2-25}$

$$\frac{x+5}{2} \times \frac{8}{x^2-25}$$

$$= \frac{8(x+5)}{2(x^2-25)}$$

$$= \frac{8\cancel{(x+5)}}{2\cancel{(x+5)}(x-5)} \quad \leftarrow \text{factorise}$$

$$= \frac{4}{x-5}$$

Note:

Without factorising the final result would not have been fully simplified.

DIVIDING ALGEBRAIC FRACTIONS

Ideas from numerical work apply to algebraic fractions.

To divide by an algebraic fraction flip the second fraction upside down and multiply.

EXAMPLE → SOLUTION

$\dfrac{2x-3}{7} \div \dfrac{6x-9}{2}$

$$\frac{2x-3}{7} \div \frac{6x-9}{2}$$

$$= \frac{2x-3}{7} \times \frac{2}{6x-9}$$

$$= \frac{2(2x-3)}{7 \times 3(2x-3)}$$

$$= \frac{2\cancel{(2x-3)}}{21\cancel{(2x-3)}}$$

$$= \frac{2}{21}$$

Note:

Without factorising the final answer would not have been fully simplified.

Fractions - Numerical Review

Exercise C: Simplifying Algebraic Fractions

Ideas from **numerical fraction work** also apply to **algebraic fractions**.

When **simplifying algebraic fractions**, you cancel **common factors** on the **numerator** and the **denominator** (these **common factors** must be **multiplied** with other **factors**!).

Simplify the following **algebraic fractions** by **cancelling common factors**.

(1) $\dfrac{a^5}{a^2}$ (2) $\dfrac{c^3}{c^7}$

(3) $\dfrac{4x^4}{6x^2}$ (4) $\dfrac{35y^9}{10y^{12}}$

Simplify the following **algebraic fractions** by **cancelling common factors**.

(5) $\dfrac{(x-1)(x+2)}{2(x+2)}$ (6) $\dfrac{a(a+5)}{7a}$

(7) $\dfrac{t+1}{4(t+1)}$ (8) $\dfrac{8x(x+3)^2}{20x^2(x+3)}$

Simplify the following **algebraic fractions** by **cancelling common factors**.
You will have to ensure that the **numerator** and **denominators** are **factorised** to spot **common factors** that will **cancel**.

(9) $\dfrac{6x+8}{9x+12}$ (10) $\dfrac{10x+15}{5x-10}$

(11) $\dfrac{8x-16}{6x+24}$ (12) $\dfrac{3x-3}{12x^2-12x}$

Simplify the following **algebraic fractions** by **cancelling common factors**.
You will have to ensure that the **numerator** and **denominators** are **factorised** to spot **common factors** that will **cancel**.

(13) $\dfrac{x^2-1}{x^2-x}$ (14) $\dfrac{4x+12}{x^2-2x-15}$

(15) $\dfrac{x^2-5x}{x^2+3x-40}$ (16) $\dfrac{x^2-9x+14}{2x^2-13x-7}$

Fractions - Numerical Review

Exercise D: Adding and Subtracting Algebraic Fractions

Ideas from **numerical fraction work** also apply to **algebraic fractions**.

When **adding** or **subtracting fractions** you need to ensure that you have a **common denominator**.

This can be found by looking for the **lowest common multiple** of the **denominators present**.

A **common multiple** can be found quickly by **multiplying the denominators** although this may not always be the **lowest** possible.

Simplify these **algebraic fractions** by writing them as a **single fraction** (complete the algebraic **addition** or **subtraction**!).

Ensure that the result is **fully simplified**.

(1) $\frac{x}{3}+\frac{x}{4}$

(2) $\frac{4x}{5}-\frac{x}{2}$

(3) $\frac{x+4}{3}+\frac{x-2}{5}$

(4) $\frac{x+4}{6}-\frac{x-2}{8}$

(5) $\frac{5x+2}{4}-\frac{3-2x}{9}$

(6) $\frac{2x+3}{3}+\frac{4x+1}{7}$

(7) $1+\frac{x}{3}$

(8) $\frac{2x}{5}-4$

(9) $\frac{2x-1}{2}+\frac{3x+5}{4}$

(10) $\frac{4x+3}{5}-\frac{8x-7}{10}$

Simplify these **algebraic fractions** by writing them as a **single fraction** (complete the algebraic **addition** or **subtraction**!).

Ensure that the result is **fully simplified**.

(11) $\frac{2}{x}+\frac{1}{3x}$

(12) $\frac{5}{4x}-\frac{2}{3x}$

(13) $\frac{5}{x-1}+\frac{2}{x+3}$

(14) $\frac{7}{3x+5}-\frac{3}{2x+1}$

(15) $\frac{x+3}{x-2}+\frac{2x}{x+5}$

(16) $\frac{x+2}{x-1}-\frac{x-3}{2x+1}$

(17) $2+\frac{5}{x}$

(18) $3-\frac{1}{2x-1}$

(19) $\frac{p}{q}+\frac{3p}{2q}$

(20) $\frac{x}{a}+\frac{2y}{5b}$

Exercise E: Multiplying and Dividing Algebraic Fractions

Ideas from **numerical fraction work** also apply to **algebraic fractions**.

When **multiplying fractions**:

- write any **mixed numbers** as **improper fractions**
- write **integers** as **themselves over one**
- **multiply** the **top numbers together** and the **bottom numbers together**

Complete these fraction multiplication questions (simplify the result if possible).

Simplify these **algebraic fractions** by writing them as a **single fraction** (complete the algebraic **multiplication**!).

Ensure that the result is **fully simplified**.

(1) $\dfrac{x+1}{3x} \times \dfrac{2x}{(x+1)^2}$ (2) $\dfrac{x^3}{(x-5)^2} \times \dfrac{(x-5)}{4x}$

(3) $\dfrac{(c+2)^2}{b^3} \times \dfrac{5b}{c+2}$ (4) $\dfrac{x+7}{12x} \times \dfrac{4x}{5}$

(5) $\dfrac{4x+8}{5} \times \dfrac{2x}{3x+6}$ (6) $\dfrac{10x-5}{2x+8} \times \dfrac{x+4}{6x-3}$

(7) $\dfrac{x^2-1}{3x} \times \dfrac{1}{x+1}$ (8) $\dfrac{3x-5x^2}{2} \times \dfrac{7x+1}{x}$

When **dividing fractions**:

- write any **mixed numbers** as **improper fractions**
- write **integers** as **themselves over one**
- **flip** the **second fraction upside down** then **multiply the fractions**

Complete these fraction division questions (simplify the result if possible).

Simplify these **algebraic fractions** by writing them as a **single fraction** (complete the algebraic **division**!).

Ensure that the result is **fully simplified**.

(9) $\dfrac{2x}{5} \div \dfrac{x}{2}$ (10) $\dfrac{4x-6}{5} \div \dfrac{2x-3}{x+1}$

(11) $\dfrac{x^2-9}{2} \div \dfrac{x-3}{4}$ (12) $\dfrac{4x}{3} \div \dfrac{x^2+2x}{2}$

FRACTIONS / ALGEBRAIC FRACTIONS — EXERCISE A

(1) $1\frac{2}{3} = \frac{5}{3}$ (2) $6\frac{1}{4} = \frac{25}{4}$ (3) $2\frac{4}{5} = \frac{14}{5}$ (4) $\frac{27}{10} = 2\frac{7}{10}$

(5) $\frac{31}{8} = 3\frac{7}{8}$ (6) $\frac{15}{2} = 7\frac{1}{2}$

(7) $\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

(8) $\frac{9}{10} - \frac{2}{3} = \frac{27}{30} - \frac{20}{30} = \frac{7}{30}$

(9) $\frac{3}{4} + \frac{5}{7} = \frac{21}{28} + \frac{20}{28} = \frac{41}{28} \; \left(1\frac{13}{28}\right)$

(10) $1\frac{1}{3} + 4\frac{1}{4} = 5\frac{4}{12} + \frac{3}{12} = 5\frac{7}{12}$

(11) $7\frac{5}{6} - 3\frac{1}{8} = 4\frac{20}{24} - \frac{3}{24} = 4\frac{17}{24}$

(12) $5\frac{2}{5} - 2\frac{2}{3} = 3\frac{6}{15} - \frac{10}{15}$

$= 3 - \frac{4}{15} = 2\frac{15}{15} - \frac{4}{15} = 2\frac{11}{15}$

(13) $\frac{2}{3} \times \frac{3}{7} = \frac{6}{21} = \frac{2}{7}$

(14) $\frac{4}{9} \times \frac{7}{10} = \frac{28}{90} = \frac{14}{45}$

(15) $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

(16) $\frac{3}{4} \times 1\frac{1}{3} = \frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

(17) $5 \times 3\frac{1}{4} = \frac{5}{1} \times \frac{13}{4} = \frac{65}{4} \; \left(16\frac{1}{4}\right)$

(18) $2\frac{3}{5} \times 4\frac{1}{9} = \frac{13}{5} \times \frac{37}{9} = \frac{481}{45} \; \left(10\frac{31}{45}\right)$

(19) $\frac{3}{10} \div \frac{1}{2} = \frac{3}{10} \times \frac{2}{1} = \frac{6}{10} = \frac{3}{5}$

(20) $\frac{7}{12} \div \frac{3}{4} = \frac{7}{12} \times \frac{4}{3} = \frac{28}{36} = \frac{7}{9}$

(21) $\frac{1}{3} \div 2 = \frac{1}{3} \div \frac{2}{1} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

(22) $2\frac{1}{2} \div 3\frac{2}{3} = \frac{5}{2} \div \frac{11}{3} = \frac{5}{2} \times \frac{3}{11} = \frac{15}{22}$

(23) $\frac{3/4}{2} = \frac{3}{4} \div 2 = \frac{3}{4} \div \frac{2}{1} = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

(24) $\frac{6}{2/3} = 6 \div \frac{2}{3} = \frac{6}{1} \div \frac{2}{3} = \frac{6}{1} \times \frac{3}{2} = \frac{18}{2} = 9$

(25) $\times 2$ (26) $\times 5$ (27) $\times \frac{3}{2}$ (28) $\times \frac{1}{2}$

(29) $\frac{4}{5} \div 2 = \frac{4}{5} \times \frac{1}{2} = \frac{4}{10} = \frac{2}{5}$

(30) $\frac{3}{7} \div 2 = \frac{3}{7} \times \frac{1}{2} = \frac{3}{14}$

(31) $2\frac{3}{4} = \frac{11}{4}$

$2\frac{3}{4} \div 2 = \frac{11}{4} \div 2 = \frac{11}{4} \times \frac{1}{2} = \frac{11}{8} = 1\frac{3}{8}$

FRACTIONS / ALGEBRAIC FRACTIONS — EXERCISE A

(32) $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(33) $\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

(34) $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$

(35) $\left(\frac{3}{10}\right)^4 = \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{81}{10000}$

(36) $\left(1\frac{2}{3}\right)^3 = \left(\frac{5}{3}\right)^3 = \frac{5}{3} \times \frac{5}{3} \times \frac{5}{3} = \frac{125}{27}$

(37) $\left(2\frac{1}{2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{5}{2} \times \frac{5}{2} = \frac{25}{4}$

FRACTIONS / ALGEBRAIC FRACTIONS — EXERCISE B

(1) (a) $y = 2(2) - 3 = 4 - 3 = 1$

(b) $y = 2(-4) - 3 = -8 - 3 = -11$

(c) $y = 2\left(\frac{1}{2}\right) - 3 = \frac{2}{2} - 3 = 1 - 3 = -2$

(d) $y = 2\left(-\frac{3}{8}\right) - 3 = -\frac{6}{8} - 3 = -3\frac{6}{8} = -3\frac{3}{4}$

(2) (a) $y = 2 - 3 = -1$

(b) $y = 2 - (-5) = 2 + 5 = 7$

(c) $y = 2 - \frac{3}{2} = \frac{4}{2} - \frac{3}{2} = \frac{1}{2}$

(d) $y = 2 - \left(-\frac{3}{2}\right) = 2 + \frac{3}{2} = \frac{4}{2} + \frac{3}{2} = \frac{7}{2} = 3\frac{1}{2}$

(3) (a) $y = 7 - \frac{3}{2}(4) = 7 - 6 = 1$

(b) $y = 7 - \frac{3}{2}(-3) = 7 - \left(-\frac{9}{2}\right) = 7 + \frac{9}{2} = 11\frac{1}{2}$

(c) $y = 7 - \frac{3}{2}\left(\frac{3}{2}\right) = 7 - \frac{9}{4} = 7 - 2\frac{1}{4} = 4\frac{3}{4}$

(d) $y = 7 - \frac{3}{2}\left(-\frac{1}{4}\right) = 7 - \left(-\frac{3}{8}\right) = 7 + \frac{3}{8} = 7\frac{3}{8}$

(4) (a) $y = (4)^2 + 2(4) = 16 + 8 = 24$

(b) $y = (-2)^2 + 2(-2) = 4 + -4 = 4 - 4 = 0$

(c) $y = \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) = \frac{1}{4} + 1 = 1\frac{1}{4}$

(d) $y = \left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right) = \frac{1}{16} - \frac{2}{4} = \frac{1}{16} - \frac{8}{16} = -\frac{7}{16}$

FRACTIONS / ALGEBRAIC FRACTIONS   EXERCISE B

(5) (a) $y = (5)^2 - 3(5) + 5 = 25 - 15 + 5 = 15$

(b) $y = (-3)^2 - 3(-3) + 5 = 9 + 9 + 5 = 23$

(c) $y = \left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right) + 5 = \frac{4}{9} - 2 + 5 = 3\frac{4}{9}$

(d) $y = \left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) + 5 = \frac{1}{4} + \frac{3}{2} + 5 = 5\frac{1}{4} + \frac{6}{4} = 5\frac{7}{4} = 6\frac{3}{4}$

(6) (a) $y = 4(2)^2 - 2 - 7 = 16 - 2 - 7 = 7$

(b) $y = 4(-1)^2 - (-1) - 7 = 4 + 1 - 7 = -2$

(c) $y = 4\left(\frac{3}{2}\right)^2 - \frac{3}{2} - 7 = \frac{36}{4} - \frac{3}{2} - 7 = 9 - 1\frac{1}{2} - 7 = \frac{1}{2}$

(d) $y = 4\left(-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right) - 7 = \frac{36}{4} + \frac{3}{2} - 7 = 9 + 1\frac{1}{2} - 7 = 3\frac{1}{2}$

(7) (a) $y = 4(5)^3 = 4 \times 125 = 500$

(b) $y = 4(-3)^3 = 4 \times -27 = -108$

(c) $y = 4\left(\frac{2}{5}\right)^3 = 4 \times \frac{8}{125} = \frac{32}{125}$

(d) $y = 4\left(-\frac{1}{4}\right)^3 = 4 \times -\frac{1}{64} = -\frac{4}{64} = \frac{-1}{16}$

(8) (a) $y = (10)^3 + 5(10)^2 - 2(10) = 1000 + 500 - 20 = 1480$

(b) $y = (-2)^3 + 5(-2)^2 - 2(-2) = -8 + 20 + 4 = 16$

(c) $y = \left(\frac{3}{4}\right)^3 + 5\left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{4}\right) = \frac{27}{64} + \frac{45}{16} - \frac{6}{4} = \frac{27}{64} + \frac{180}{64} - \frac{96}{64} = \frac{111}{64}$

(d) $y = \left(-\frac{1}{2}\right)^3 + 5\left(\frac{-1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) = -\frac{1}{8} + \frac{5}{4} + 1 = -\frac{1}{8} + \frac{10}{8} + \frac{8}{8} = \frac{17}{8}$

FRACTIONS / ALGEBRAIC FRACTIONS — EXERCISE C

① $\frac{a^5}{a^2} = a^3$ ② $\frac{c^3}{c^7} = \frac{1}{c^4}$ ③ $\frac{4x^4}{6x^2} = \frac{2x^2}{3}$ ④ $\frac{35y^9}{10y^{12}} = \frac{7}{2y^3}$

⑤ $\frac{(x-1)\cancel{(x+2)}}{2\cancel{(x+2)}} = \frac{x-1}{2}$ ⑥ $\frac{\cancel{a}(a+5)}{7\cancel{a}} = \frac{(a+5)}{7}$

⑦ $\frac{\cancel{(t+1)}}{4\cancel{(t+1)}} = \frac{1}{4}$ ⑧ $\frac{8x(x+3)^2}{20x^2(x+3)} = \frac{8\cancel{x}(x+3)\cancel{(x+3)}}{20\cancel{x}x\cancel{(x+3)}} = \frac{2(x+3)}{5x}$

⑨ $\frac{6x+8}{9x+12} = \frac{2\cancel{(3x+4)}}{3\cancel{(3x+4)}} = \frac{2}{3}$ ⑩ $\frac{10x+15}{5x-10} = \frac{\cancel{5}(2x+3)}{\cancel{5}(x-2)} = \frac{2x+3}{x-2}$

⑪ $\frac{8x-16}{6x+24} = \frac{8(x-2)}{6(x+4)} = \frac{4(x-2)}{3(x+4)}$ ⑫ $\frac{3x-3}{12x^2-12x} = \frac{3\cancel{(x-1)}}{12x\cancel{(x-1)}} = \frac{1}{4x}$

⑬ $\frac{x^2-1}{x^2-x} = \frac{(x+1)(x-1)}{x(x-1)}$

$= \frac{x+1}{x}$

⑭ $\frac{4x+12}{x^2-2x-15} = \frac{4(x+3)}{(x-5)(x+3)}$

$= \frac{4}{x-5}$

⑮ $\frac{x^2-5x}{x^2+3x-40} = \frac{x(x-5)}{(x+8)(x-5)}$

$= \frac{x}{x+8}$

⑯ $\frac{x^2-9x+14}{2x^2-13x-7} = \frac{(x-7)(x-2)}{(x-7)(2x+1)}$

$= \frac{x-2}{2x+1}$

FRACTIONS / ALGEBRAIC FRACTIONS — EXERCISE D

① $\frac{x}{3} + \frac{x}{4} = \frac{4x}{12} + \frac{3x}{12} = \frac{7x}{12}$

② $\frac{4x}{5} - \frac{x}{2} = \frac{8x}{10} - \frac{5x}{10} = \frac{3x}{10}$

③ $\frac{x+4}{3} + \frac{x-2}{5} = \frac{5(x+4)}{15} + \frac{3(x-2)}{15} = \frac{5x+20+3x-6}{15} = \frac{8x+14}{15}$

④ $\frac{x+4}{6} - \frac{x-2}{8} = \frac{4(x+4)}{24} - \frac{3(x-2)}{24} = \frac{4x+16-3x+6}{24} = \frac{x+22}{24}$

FRACTIONS / ALGEBRAIC FRACTIONS — EXERCISE D

(5) $\dfrac{5x+2}{4} - \dfrac{3-2x}{9} = \dfrac{9(5x+2)}{36} - \dfrac{4(3-2x)}{36} = \dfrac{45x+18-12+8x}{36} = \dfrac{53x+6}{36}$

(6) $\dfrac{2x+3}{3} + \dfrac{4x+1}{7} = \dfrac{7(2x+3)}{21} + \dfrac{3(4x+1)}{21} = \dfrac{14x+21+12x+3}{21} = \dfrac{26x+24}{21}$

(7) $1 + \dfrac{x}{3} = \dfrac{3}{3} + \dfrac{x}{3} = \dfrac{3+x}{3}$

(8) $\dfrac{2x}{5} - 4 = \dfrac{2x}{5} - \dfrac{20}{5} = \dfrac{2x-20}{5} = \dfrac{2(x-10)}{5}$

(9) $\dfrac{2x-1}{2} + \dfrac{3x+5}{4} = \dfrac{2(2x-1)}{4} + \dfrac{3x+5}{4} = \dfrac{4x-2+3x+5}{4} = \dfrac{7x+3}{4}$

(10) $\dfrac{4x+3}{5} - \dfrac{8x-7}{10} = \dfrac{2(4x+3)}{10} - \dfrac{(8x-7)}{10} = \dfrac{8x+6-8x+7}{10} = \dfrac{13}{10}$

(11) $\dfrac{2}{x} + \dfrac{1}{3x} = \dfrac{6}{3x} + \dfrac{1}{3x} = \dfrac{6+1}{3x} = \dfrac{7}{3x}$

(12) $\dfrac{5}{4x} - \dfrac{2}{3x} = \dfrac{15}{12x} - \dfrac{8}{12x} = \dfrac{7}{12x}$

(13) $\dfrac{5}{(x-1)} + \dfrac{2}{(x+3)} = \dfrac{5(x+3)}{(x-1)(x+3)} + \dfrac{2(x-1)}{(x-1)(x+3)} = \dfrac{5x+15+2x-2}{(x-1)(x+3)}$

$= \dfrac{7x+13}{(x-1)(x+3)}$

(14) $\dfrac{7}{(3x+5)} - \dfrac{3}{(2x+1)} = \dfrac{7(2x+1)-3(3x+5)}{(3x+5)(2x+1)} = \dfrac{14x+7-9x-15}{(3x+5)(2x+1)} = \dfrac{5x-8}{(3x+5)(2x+1)}$

(15) $\dfrac{x+3}{x-2} + \dfrac{2x}{x+5} = \dfrac{(x+3)(x+5)+2x(x-2)}{(x-2)(x+5)} = \dfrac{x^2+8x+15+2x^2-4x}{(x-2)(x+5)} = \dfrac{3x^2+4x+15}{(x-2)(x+5)}$

(16) $\dfrac{x+2}{x-1} - \dfrac{x-3}{2x+1} = \dfrac{(x+2)(2x+1)-(x-3)(x-1)}{(x-1)(2x+1)} = \dfrac{2x^2+5x+2-(x^2-4x+3)}{(x-1)(2x+1)} = \dfrac{x^2+9x-1}{(x-1)(2x+1)}$

FRACTIONS / ALGEBRAIC FRACTIONS — EXERCISE D

(17) $2+\frac{5}{x} = \frac{2x}{x}+\frac{5}{x} = \frac{2x+5}{x}$

(18) $3-\frac{1}{2x-1} = \frac{3(2x-1)}{2x-1}-\frac{1}{2x-1} = \frac{6x-3-1}{2x-1} = \frac{6x-4}{2x-1}$

$= \frac{2(3x-2)}{2x-1}$

(19) $\frac{p}{q}+\frac{3p}{2q} = \frac{2p}{2q}+\frac{3p}{2q} = \frac{5p}{2q}$

(20) $\frac{x}{a}+\frac{2y}{5b} = \frac{5bx}{5ab}+\frac{2ay}{5ab} = \frac{5bx+2ay}{5ab}$

FRACTIONS / ALGEBRAIC FRACTIONS — EXERCISE E

(1) $\frac{2x(x+1)}{3x(x+1)^2} = \frac{2}{3(x+1)}$

(2) $\frac{x^3(x-5)}{4x(x-5)^2} = \frac{x^2}{4(x-5)}$

(3) $\frac{5b(c+2)^2}{b^3(c+2)} = \frac{5(c+2)}{b^2}$

(4) $\frac{4x(x+7)}{60x} = \frac{x+7}{15}$

(5) $\frac{2x(4x+8)}{5(3x+6)} = \frac{2x\times 4(x+2)}{5\times 3(x+2)} = \frac{8x(x+2)}{15(x+2)} = \frac{8x}{15}$

(6) $\frac{(10x-5)(x+4)}{(2x+8)(6x-3)} = \frac{5(2x-1)(x+4)}{2(x+4)\,3(2x-1)} = \frac{5(x+4)(2x-1)}{6(x+4)(2x-1)} = \frac{5}{6}$

(7) $\frac{x^2-1}{3x(x+1)} = \frac{(x+1)(x-1)}{3x(x+1)} = \frac{x-1}{3x}$

(8) $\frac{(3x-5x^2)(7x+1)}{2x} = \frac{x(3-5x)(7x+1)}{2x} = \frac{(3-5x)(7x+1)}{2}$

FRACTIONS / ALGEBRAIC FRACTIONS EXERCISE E

(9) $\frac{2x}{5} \times \frac{2}{x} = \frac{4x}{5x} = \frac{4}{5}$

(10) $\frac{4x-6}{5} \times \frac{x+1}{2x-3} = \frac{2(2x-3)(x+1)}{5(2x-3)} = \frac{2(x+1)}{5}$

(11) $\frac{x^2-9}{2} \times \frac{4}{x-3} = \frac{4(x+3)(x-3)}{2(x-3)} = 2(x+3)$

(12) $\frac{4x}{3} \times \frac{2}{x^2+2x} = \frac{8x}{3x(x+2)} = \frac{8}{3(x+2)}$

INDICES

Indices

Objectives

- ★ know the meaning of positive indices
- ★ know that any number to the power zero yields one (zero index)
- ★ know the meaning of negative indices (give reciprocals)
- ★ know the meaning of fractional indices (give roots)
- ★ evaluate numbers to various powers (both negative and fractional) without a calculator
- ★ know the laws of indices
- ★ simplify algebraic expressions involving indices
- ★ write expressions in index form

REFERENCE IN OTHER RESOURCES

Head Start: Laws of Indices - Pages 6-10.

Alpha Workbooks: Indices - Pages 5-6.

Indices - Number

INDICES AND NUMBER

MEANING (BASIC)

Indices are an efficient way to express the repeated multiplication of a term by itself:

$$\boxed{b^n = b \times b \times \ldots \times b}$$ {b appears n-times}

EXAMPLE

Evaluate 6^2

SOLUTION

$6^2 = 6 \times 6 = 36$

EXAMPLE

Evaluate 2^6

SOLUTION

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

THE ZERO INDEX

Any number to the power zero yields one.

$$\boxed{b^0 = 1}$$

EXAMPLE

Evaluate 3^0

SOLUTION

$3^0 = 1$

EXAMPLE

Work out $0{\cdot}2^0$

SOLUTION

$0{\cdot}2^0 = 1$

INDICES AND NUMBER

NEGATIVE INDICES

Negative indices give reciprocals.

The reciprocal of a number is $\dfrac{1}{\text{the number}}$.

To find the reciprocal of a fraction, flip the fraction upside down.

$$\boxed{b^{-n} = \frac{1}{b^n}} \qquad \boxed{\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n}$$

EXAMPLE

Evaluate 2^{-4}

SOLUTION

$2^{-4} = \dfrac{1}{2^4} = \dfrac{1}{16}$

EXAMPLE

Evaluate 5^{-2}

SOLUTION

$5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

EXAMPLE

Evaluate 3^{-1}

SOLUTION

$3^{-1} = \dfrac{1}{3^1} = \dfrac{1}{3}$

EXAMPLE

Evaluate $\left(\frac{1}{2}\right)^{-3}$

SOLUTION

$\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3 = (2)^3 = 8$

EXAMPLE

Evaluate $\left(\frac{2}{3}\right)^{-2}$

SOLUTION

$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \dfrac{9}{4}$

INDICES AND NUMBER

THE POWER FLOWER

FRACTIONAL INDICES

Fractional indices give roots.

- Power $\frac{1}{2}$ 2ND or square root $b^{1/2} = \sqrt{b}$
- Power $\frac{1}{3}$ 3RD or cube root $b^{1/3} = \sqrt[3]{b}$
- Power $\frac{1}{4}$ 4TH root $b^{1/4} = \sqrt[4]{b}$
- Power $\frac{1}{n}$ nTH root $\boxed{b^{1/n} = \sqrt[n]{b}}$

EXAMPLE

Evaluate $49^{1/2}$

SOLUTION

$49^{1/2} = \sqrt{49} = 7$

EXAMPLE

Evaluate $8^{1/3}$

SOLUTION

$8^{1/3} = \sqrt[3]{8} = 2$

EXAMPLE

$1000000^{1/6} = ?$

SOLUTION

$1000000^{1/6} = \sqrt[6]{1000000} = 10$

EXAMPLE

Evaluate $64^{1/3}$

SOLUTION

$64^{1/3} = \sqrt[3]{64} = 4$

INDICES AND NUMBER

MIXED INDICES

ROOT FIRST

EXAMPLE

Evaluate $27^{2/3}$

SOLUTION

$27^{2/3}$

SQUARE (LAST)

CUBE ROOT (FIRST)

$27^{2/3} = 3^2 = 9$

EXAMPLE

Evaluate $16^{-3/4}$

SOLUTION

$16^{-3/4}$

CUBE-RECIPROCAL (LAST)

FOURTH ROOT (FIRST)

$16^{-3/4} = 2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

$$\boxed{b^{m/n} = \left(\sqrt[n]{b}\right)^m}$$

nTH ROOT THEN TO POWER m

Indices - Number

Exercise A: Evaluating Numerical Expressions

Recall the basic meaning of **indices**:

$$b^n = b \times b \times b \times .... \times b \qquad \text{the } b \text{ appears } n\text{-times}$$

Evaluate the following without a calculator:

(1) 9^2	(2) 5^3	(3) 2^5	(4) 10^4
(5) 1^{10}	(6) 3^4	(7) 7^2	(8) 2^7

Recall that **any number** to the **power zero** gives **one**.

Evaluate the following without a calculator:

(9) 7^0	(10) 2^0	(11) 0.3^0	(12) 99^0

Recall that **a negative power** gives a **reciprocal**:

$$b^{-n} = \frac{1}{b^n}$$

Evaluate the following without a calculator:

(13) 7^{-1}	(14) 3^{-2}	(15) 5^{-3}	(16) 10^{-5}
(17) 4^{-3}	(18) 9^{-1}	(19) 2^{-6}	(20) 8^{-2}

Recall that **fractional powers** gives a **roots**:

$b^{\frac{1}{2}} = \sqrt{b}$ **power half** is **square root** (**2** on **bottom** in **power**)

$b^{\frac{1}{3}} = \sqrt[3]{b}$ **power third** is **cube root** (**3** on **bottom** in **power**)

$b^{\frac{1}{4}} = \sqrt[4]{b}$ **power quarter** is **fourth root** (**4** on **bottom** in **power**)

$b^{\frac{1}{n}} = \sqrt[n]{b}$ **power 1/*n*** is ***n*th root** (***n*** on **bottom** in **power**)

Evaluate the following without a calculator:

(21) $25^{\frac{1}{2}}$	(22) $81^{\frac{1}{2}}$	(23) $144^{\frac{1}{2}}$	(24) $36^{\frac{1}{2}}$
(25) $8^{\frac{1}{3}}$	(26) $1000^{\frac{1}{3}}$	(27) $27^{\frac{1}{3}}$	(28) $125^{\frac{1}{3}}$
(29) $32^{\frac{1}{5}}$	(30) $81^{\frac{1}{4}}$	(31) $100000^{\frac{1}{5}}$	(32) $1^{\frac{1}{7}}$

Indices - Number

Exercise A: Evaluating Numerical Expressions

When a **more general** fraction **appears** as the **power**:

- consider the **root first**
- then do the **other bit!**

Evaluate the following without a calculator:

(33) $8^{\frac{2}{3}}$ (34) $32^{\frac{3}{5}}$ (35) $1000^{\frac{4}{3}}$ (36) $125^{\frac{2}{3}}$

(37) $27^{\frac{4}{3}}$ (38) $16^{\frac{7}{4}}$ (39) $400^{\frac{3}{2}}$ (40) $81^{\frac{3}{4}}$

(41) $16^{-\frac{3}{4}}$ (42) $1000^{-\frac{2}{3}}$ (43) $25^{-\frac{3}{2}}$ (44) $32^{-\frac{2}{5}}$

Evaluate the following without a calculator:

(45) $\left(\frac{1}{2}\right)^{-1}$ (46) $\left(\frac{3}{4}\right)^{-1}$ (47) $\left(\frac{1}{5}\right)^{-3}$ (48) $\left(\frac{3}{2}\right)^{-4}$

(49) $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$ (50) $\left(\frac{125}{8}\right)^{-\frac{2}{3}}$

Check a range of these answers using a calculator to ensure you know how to use the **power key** and deal with **fractional powers**.

P. Leary

Indices - Algebra

INDICES AND ALGEBRA

$b^n \times b^m = b^{n+m}$	MULTIPLY (ADD POWERS)
$b^n \div b^m = b^{n-m}$	DIVIDE (SUBTRACT POWERS)
$(b^n)^m = b^{n \times m}$	POWER TO POWER (MULTIPLY POWERS)
$(a \times b)^m = a^m \times b^m$	BRACKET RULE (ATTACH POWER TO EACH)

EXAMPLE

Find the missing power $\sqrt[3]{n} = n^?$

SOLUTION

$\sqrt[3]{n} = n^{1/3}$

EXAMPLE

Find the missing power $\frac{1}{n^4} = n^?$

SOLUTION

$\frac{1}{n^4} = n^{-4}$

INDICES AND ALGEBRA

EXAMPLE

Simplify $3t^2 \times 5t^4$

SOLUTION

$3t^2 \times 5t^4 = 15t^6$

multiply numbers
add powers on letter

EXAMPLE

Simplify $(2ab^2)^3$

SOLUTION

$(2ab^2)^3 = 2^3 a^3 b^6 = 8a^3b^6$

attach power 3 (cube) all terms in bracket

EXAMPLE

Simplify $(5x^2y^3)^2 \times 4xy^{-1}$

SOLUTION

$(5x^2y^3)^2 \times 4xy^{-1}$

$= 5^2x^4y^6 \times 4xy^{-1}$

$= 25x^4y^6 \times 4xy^{-1}$

$= 100x^5y^5$

square all terms in bracket
multiply numbers
add powers on letters

INDICES AND ALGEBRA

EXAMPLE

Simplify $\frac{12a^7b^5}{3a^2b}$

SOLUTION

$\frac{12a^7b^5}{3a^2b} = 4a^5b^4$

divide numbers
subtract powers on letters

EXAMPLE

Simplify $\frac{2x^2y^7}{10x^4y^4}$

SOLUTION

$\frac{2x^2y^7}{10x^4y^4} = \frac{x^{-2}y^3}{5} \quad \left(= \frac{y^3}{5x^2}\right)$

EXAMPLE

Simplify $\sqrt{9ab^3} \times a \times \sqrt[3]{b}$

SOLUTION

$\sqrt{9ab^3} \times a \times \sqrt[3]{b}$

$= (9ab^3)^{1/2} \times a \times b^{1/3}$

$= 3a^{1/2}b^{3/2} \times a \times b^{1/3}$

$= 3a^{3/2}b^{11/6}$

$\frac{1}{2} + 1 = \frac{1}{2} + \frac{2}{2} = \frac{3}{2}$

$\frac{3}{2} + \frac{1}{3} = \frac{9}{6} + \frac{2}{6} = \frac{11}{6}$

Indices - Algebra

Exercise B: Simplifying Algebraic Expressions

Know the **laws of indices**:

The **multiplication law**:	$b^n \times b^m = b^{m+n}$
The **division law**:	$b^n \div b^m = b^{m-n}$
The **power-to-power law**:	$(b^n)^m = b^{n\times m}$
The **bracket law**:	$(a\times b)^n = a^n \times b^n$

Find the missing power *i.e.* $x^?$.

(1) $\sqrt{x} = x^?$

(2) $\dfrac{1}{x^3} = x^?$

(3) $\sqrt[5]{x} = x^?$

(4) $\sqrt{x^7} = x^?$

(5) $\left(\sqrt[3]{x}\right)^4$

(6) $\dfrac{1}{\sqrt{x}}$

Write the following in **index form** *i.e.* $n^?$.

(7) the square root of n

(8) the reciprocal of n^2

(9) $\sqrt[4]{n}$

(10) $\sqrt[3]{n^2}$

(11) $\dfrac{1}{n^3}$

(12) $\dfrac{1}{\sqrt{n^3}}$

Simplify the following **algebraic expressions**.

(13) $2x^2 \times 3x^5$

(14) $(2c)^3$

(15) $2a^2b \times 5a^3b^5$

(16) $7p^5q^3 \times 3p^3q^{-1}$

(17) $(10xy^2)^3$

(18) $(5a^3b^2)^2 \times (a^2b)^4$

(19) $(2d^{-2})^{-1}$

(20) $(2v^2w)^3 \times (3vw^3)^2$

Indices - Algebra

Exercise B: Simplifying Algebraic Expressions

Know the **laws of indices**:

The **multiplication law**:	$b^n \times b^m = b^{m+n}$
The **division law**:	$b^n \div b^m = b^{m-n}$
The **power-to-power law**:	$(b^n)^m = b^{n\times m}$
The **bracket law**:	$(a\times b)^n = a^n \times b^n$

Simplify the following **algebraic expressions** involving **reciprocals**.

(21) $\dfrac{10a^7}{15a^2}$

(22) $\dfrac{8p^5q^2}{2p^2q}$

(23) $\dfrac{4x^3y^4}{8x^3y}$

(24) $\dfrac{9m^2}{3m^3}$

(25) $\dfrac{(2p^2q)^3}{(3pq)^4}$

(26) $\dfrac{9a^2b^5c^2}{12a^5bc^{-2}}$

Simplify the following **algebraic expressions** involving **roots**.

(27) $x\sqrt{x}$

(28) $\dfrac{c}{\sqrt{c}}$

(29) $(\sqrt{2ab^3})^2$

(30) $\sqrt{p}\times\sqrt[3]{p}$

(31) $\dfrac{1}{\sqrt[3]{q}}\times\sqrt{q}$

(32) $\sqrt{x^2y}\times 3x^3\sqrt{y}$

INDICES — EXERCISE A

(1) $9^2 = 9\times9 = 81$

(2) $5^3 = 5\times5\times5 = 125$

(3) $2^5 = 2\times2\times2\times2\times2 = 32$

(4) $10^4 = 10\times10\times10\times10 = 10000$

(5) $1^{10} = 1$

(6) $3^4 = 3\times3\times3\times3 = 81$

(7) $7^2 = 7\times7 = 49$

(8) $2^7 = 2\times2\times2\times2\times2\times2\times2 = 128$

(9) $7^0 = 1$

(10) $2^0 = 1$

(11) $0.3^0 = 1$

(12) $99^0 = 1$

(13) $7^{-1} = \frac{1}{7^1} = \frac{1}{7}$

(14) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(15) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

(16) $10^{-5} = \frac{1}{10^5} = \frac{1}{100000}$

(17) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

(18) $9^{-1} = \frac{1}{9^1} = \frac{1}{9}$

(19) $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$

(20) $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

(21) $25^{\frac{1}{2}} = \sqrt{25} = 5$

(22) $81^{\frac{1}{2}} = \sqrt{81} = 9$

(23) $144^{1/2} = \sqrt{144} = 12$

(24) $36^{1/2} = \sqrt{36} = 6$

(25) $8^{1/3} = \sqrt[3]{8} = 2$

(26) $1000^{1/3} = \sqrt[3]{1000} = 10$

(27) $27^{1/3} = \sqrt[3]{27} = 3$

(28) $125^{1/3} = \sqrt[3]{125} = 5$

(29) $32^{1/5} = \sqrt[5]{32} = 2$

(30) $81^{1/4} = \sqrt[4]{81} = 3$

(31) $100000^{1/5} = \sqrt[5]{100000} = 10$

(32) $1^{1/7} = \sqrt[7]{1} = 1$

(33) $8^{2/3} = 2^2 = 4$

(34) $32^{3/5} = 2^3 = 8$

(35) $10000^{4/3} = 10^4 = 10000$

(36) $125^{2/3} = 5^2 = 25$

(37) $27^{4/3} = 3^4 = 81$

(38) $16^{7/4} = 2^7 = 128$

(39) $400^{3/2} = 20^3 = 8000$

(40) $81^{3/4} = 3^3 = 27$

(41) $16^{-3/4} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(42) $1000^{-2/3} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

(43) $25^{-3/2} = 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

(44) $32^{-2/5} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

(45) $\left(\frac{1}{2}\right)^{-1} = \frac{2}{1} = 2$

(46) $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

(47) $\left(\frac{1}{5}\right)^{-3} = \left(\frac{5}{1}\right)^3 = 125$

(48) $\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

(49) $\left(\frac{9}{16}\right)^{-1/2} = \left(\frac{16}{9}\right)^{1/2} = \frac{4}{3}$

(50) $\left(\frac{125}{8}\right)^{-2/3} = \frac{4}{25}$

INDICES

EXERCISE B

(1) $\sqrt{x} = x^{1/2}$

(2) $\frac{1}{x^3} = x^{-3}$

(3) $\sqrt[5]{x} = x^{1/5}$

(4) $\sqrt{x^7} = x^{7/2}$

(5) $(\sqrt[3]{x})^4 = x^{4/3}$

(6) $\frac{1}{\sqrt{x}} = x^{-1/2}$

(7) $\sqrt{n} = n^{1/2}$

(8) $\frac{1}{n^2} = n^{-2}$

(9) $\sqrt[4]{n} = n^{1/4}$

(10) $\sqrt[3]{n^2} = n^{2/3}$

(11) $\frac{1}{n^3} = n^{-3}$

(12) $\frac{1}{\sqrt{n^3}} = n^{-3/2}$

(13) $2x^2 \times 3x^5$
$= 6x^7$

(14) $(2c)^3$
$= 2^3c^3$
$= 8c^3$

(15) $2a^2b \times 5a^3b^5$
$= 10a^5b^6$

(16) $7p^5q^3 \times 3p^3q^{-1}$
$= 21p^8q^2$

(17) $(10xy^2)^3$
$= 10^3x^3y^6$
$= 1000x^3y^6$

(18) $(5a^3b^2)^2 \times (a^2b)^4$
$= 5^2a^6b^4 \times a^8b^4$
$= 25a^{14}b^8$

(19) $(2d^{-2})^{-1}$
$= 2^{-1}d^2$
$= \frac{1}{2}d^2$

(20) $(2v^2w)^3 \times (3vw^3)^2$
$= 2^3v^6w^3 \times 3^2v^2w^6$
$= 8v^6w^3 \times 9v^2w^6$
$= 72v^8w^9$

(21) $\frac{10a^7}{15a^2} = \frac{2a^5}{3}$

(22) $\frac{8p^5q^2}{2p^2q} = 4p^3q$

(23) $\frac{4x^3y^4}{8x^3y} = \frac{x^0y^3}{2} = \frac{y^3}{2}$

(24) $\frac{9m^2}{3m^3} = 3m^{-1}$ or $\frac{3}{m}$

(25) $\frac{(2p^2q)^3}{(3pq)^4} = \frac{8p^6q^3}{81p^4q^4}$
$= \frac{8p^2}{81q}$

(26) $\frac{9a^2b^5c^2}{12a^5bc^{-2}}$
$= \frac{3b^4c^4}{4a^3}$

(27) $x\sqrt{x} = x^1 \times x^{1/2} = x^{3/2}$

(28) $\frac{c}{\sqrt{c}} = \frac{c^1}{c^{1/2}} = c^{1/2}$

(29) $(\sqrt{2ab^3})^2$
$= 2ab^3$

(30) $\sqrt{p} \times \sqrt[3]{p} = p^{1/2} \times p^{1/3} = p^{5/6}$

(31) $\frac{1}{\sqrt[3]{q}} \times \sqrt{q} = q^{-1/3} \times q^{1/2}$
$= q^{1/6}$

(32) $\sqrt{x^2y} \times 3x^3\sqrt{y}$
$= xy^{1/2} \times 3x^3y^{1/2}$
$= 3x^4y$

TRIGONOMETRY

Non-Right Angled Triangles

Objectives

★ know how to do trigonometry with right angled triangles

- ★ meaning of ratios sin, cos and tan (SOHCAHTOA)
- ★ Identify the correct ratio: sin, cos and tan (SOHCAHTOA), to use in a problem
- ★ use sin, cos or tan keys to find missing sides given an angle and a side
- ★ use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ keys to find a missing angle given two sides
- ★ solve problems in context using trigonometry, including bearings

★ know and use the sine rule for non-right angled triangles

- ☆ identify when the sine rule can be used (given an angle and side opposite)
- ☆ use the sine rule to find missing angles in triangles ($\sin^{-1}$ key)
- ☆ use the sine rule to find missing sides in triangles (sin key)
- ☆ solve problems in context using the sine rule, including bearings

★ know and use the cosine rule for non-right angled triangles

- ☆ identify when the cosine rule can be used (not the sine rule!)
- ☆ use the cosine rule to find missing angles in triangles ($\cos^{-1}$ key)
- ☆ use the cosine rule to find missing sides in triangles (cos key)
- ☆ solve problems in context using the sine rule, including bearings

★ know and use the area formula for non-right angled triangles

REFERENCE IN OTHER RESOURCES

Head Start: Topic not covered.

Alpha Workbooks: Basic trigonometry - Pages 29-34; Sine & Cosine Rules 35-38.

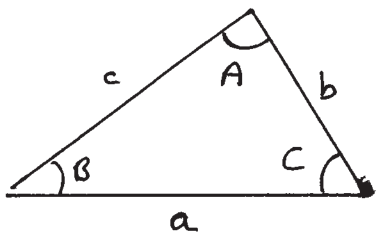
Trigonometry (Non-Right Angled Triangles) - Labelling Triangles

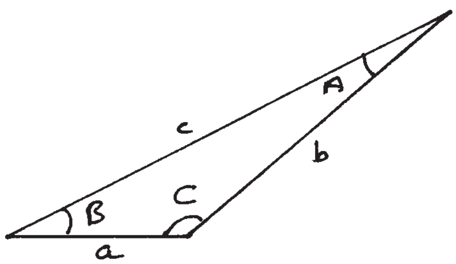
TRIGONOMETRY NON-RIGHT-ANGLED TRIANGLES

NOTATION

There are three key trigonometric results to learn for non-right angled triangles.

All relate to a triangle like the ones shown (basically any triangle, including ones with obtuse angles).





Note

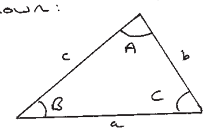
- Angles are marked using capital letters
- Sides are labelled using lower case letters
  (side a is opposite angle A,
  side b is opposite angle B,
  side c is opposite angle C)
- Other letters can also be used e.g. P, Q and R for angles with p, q and r for sides respectively
- As you will see, each formula gives you a bargain three for the price of one as other formulae can be generated by interchanging letters in a symmetric way.

Trigonometry (Non-Right Angled Triangles) - The Sine Rule

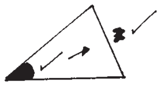
TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

THE SINE RULE

The sine rule is used to calculate missing sides (or angles) in non-right angled triangles like the one shown:



The sine rule should be used when you know an angle and the length of the side opposite this, along with any other given information.



MUST KNOW AN ANGLE AND LENGTH OF SIDE OPPOSITE IT TO USE THE SINE RULE (AT LEAST ONE OTHER PIECE OF INFORMATION WILL ALSO BE GIVEN).

The sine rule states that:

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

This is really three formulae:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the appropriate one for the problem you are solving.

Again, different letters may also be used.

TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

THE SINE RULE

When calculating angles rather than sides you might find it easier to use the 'upside-down' form (or the 'reciprocal' form) of the sine rule.

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

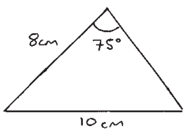
In any problem, once two angles in a triangle are known, the fact that angles in a triangle sum to 180° can be used to find the third angle.

TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

THE SINE RULE

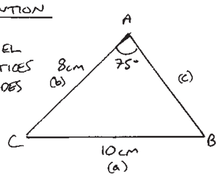
EXAMPLE

Given this triangle, determine all of the missing angles.



SOLUTION

LABEL VERTICES & SIDES



DETERMINE ANGLE B FIRST AS SIDE b IS KNOWN

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{8} = \frac{\sin 75}{10}$$

$$\sin B = \frac{8 \times \sin 75}{10}$$

$$\sin B = 0.772741$$

$$B = \sin^{-1}(0.772741) = 50.6°$$

NOW USE ANGLES IN A TRIANGLE SUM TO 180° TO FIND C (SIDE c DOES NOT HAVE TO BE DETERMINED).

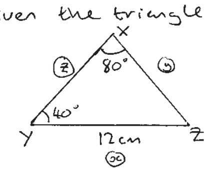
$$C = 180 - 75 - 50.6$$

$$C = 54.4°$$

TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

EXAMPLE

Given the triangle



Find the size of angle Z and length of the side z.

SOLUTION

Angle Z:

using angles in a triangle

$$Z = 180° - 80° - 40° = 60°$$

Side z:

use the sine rule in the form

$$\frac{z}{\sin Z} = \frac{x}{\sin X}$$

Since only one unknown!

$$\frac{z}{\sin 60} = \frac{12}{\sin 80}$$

$$z = \frac{12 \times \sin 60}{\sin 80}$$

$$z = 10.6\text{ cm}$$

Trigonometry (Non-Right Angled Triangles) - The Sine Rule

Exercise A: The Sine Rule

The **sine rule** can be used to **find missing angles** and **sides** in **non-right angled triangles** (use the **sine rule** if you know an **angle** and the **side opposite**).

To avoid too much algebra, use the form with the **unknown** on the **top**.

The two forms of the sine rule are:

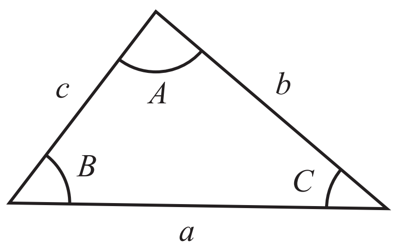
Angle Form

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Side Form

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

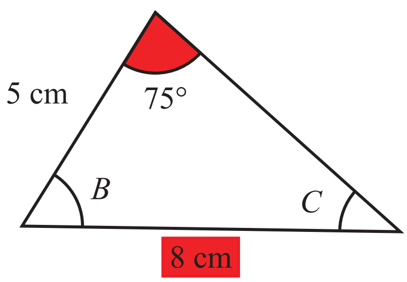
The triangle is labelled up in the usual way (other letters may be used):



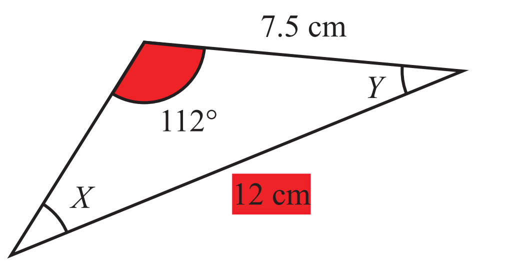
Note: other letters may be used, the structure of the formula is the same (use two bits!).

Use the **sine rule** to find all the **missing angles** in these triangles:

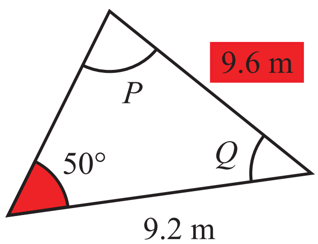
(1)



(2)



(3)



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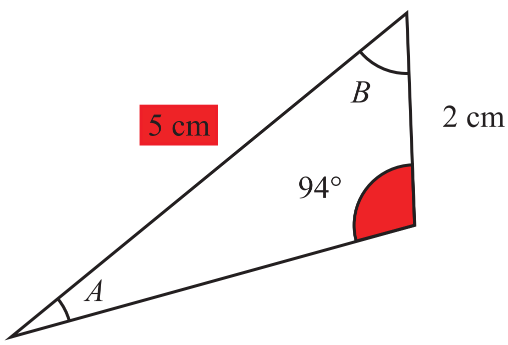
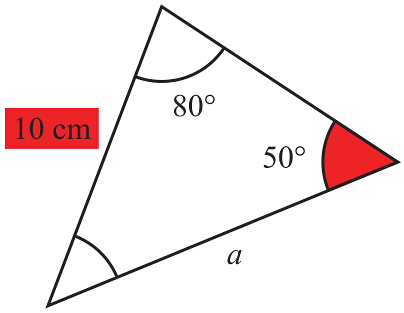


Trigonometry (Non-Right Angled Triangles) - The Sine Rule

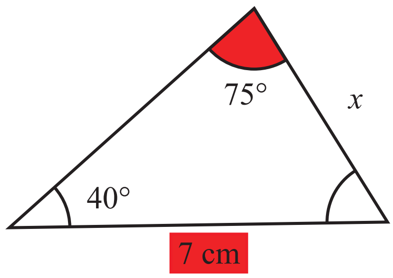
<u>Exercise A:</u> The Sine Rule

Use the **sine rule** to find the labeled **missing side** in these triangles.

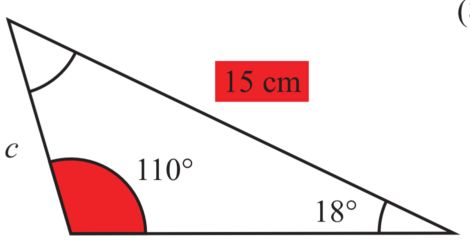
(5)



(6)



(7)



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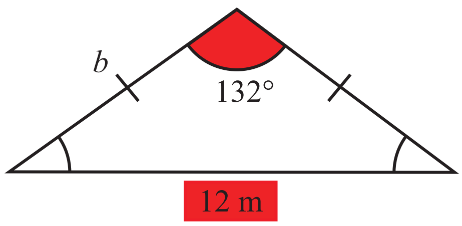
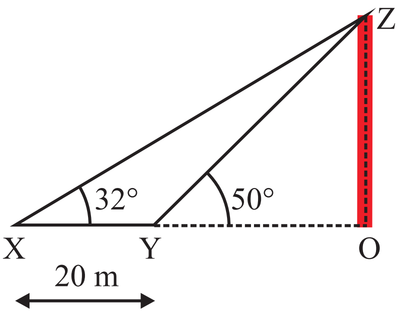


(9) The diagram shows an areal mast.

Billy measures the angle of elevation to the top from X, it is 32°.

He then walks 20 m towards the tower (Y) and measures the angle of elevation now to be 50°.



(a) use basic angle rules to calculate the angles XYZ and XZY

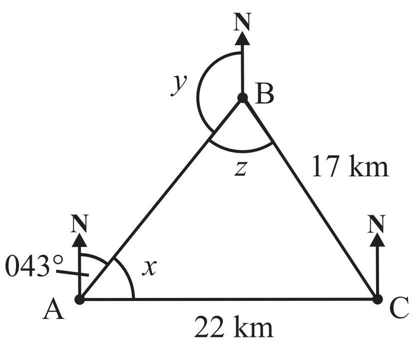
(b) use the sine rule to calculate the side YZ

(c) hence use basic trigonometry to calculate the height of the mast OZ

(10) The diagram shows three reefs A, B and C.

Distance BC is 17 km and distance AC is 22 km; C is due East of A.

B is on a bearing of 043° from A.



(a) explain why angle x is 47° and angle y is 137°?

(b) calculate the angle marked z

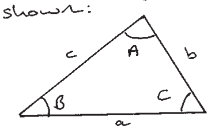
(c) what is the bearing of C from B?

Trigonometry (Non-Right Angled Triangles) - The Cosine Rule

TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

THE COSINE RULE

The cosine rule is used to calculate missing sides (or angles) in non-right angled triangles like the one shown:



The cosine rule should be used when you can't use the sine rule.

This could be when you know two sides and the included angle ~~or~~ when you know all three sides but no angles.

The cosine rule states that:

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

Note

- side a at front
- angle A at back

} same letter front and back

- lengths b and c present in middle.

Again, other symmetric formula can be generated quickly by swapping letters in a systematic way (just ensure that the above points are adhered to).

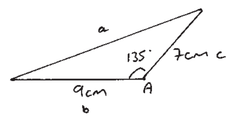
$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$

TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

THE COSINE RULE

EXAMPLE

Find the length of the third side of a triangle with sides 9cm and 7cm and an included angle of 135° as shown.



The appropriate form is:

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 9^2 + 7^2 - 2 \times 9 \times 7 \times \cos 135$

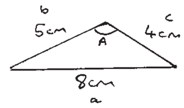
$a^2 = 130 - 126\cos 135$

$a^2 = 219 \cdot 1$

$a = 14 \cdot 8\text{cm}$

EXAMPLE

Calculate the largest angle in a triangle with sides of 4cm, 5cm and 8cm.



Label up as shown (or otherwise!)

The appropriate form is:

$a^2 = b^2 + c^2 - 2bc\cos A$

substituting

$8^2 = 5^2 + 4^2 - 2 \times 5 \times 4 \times \cos A$

$64 = 41 - 40\cos A$

re-arranging

$40\cos A = 41 - 64$

$40\cos A = -23$

$\cos A = \frac{-23}{40}$

$A = \cos^{-1}\left(\frac{-23}{40}\right)$

$A = 125°$

Trigonometry (Non-Right Angled Triangles) - The Cosine Rule

Exercise B: The Cosine Rule

The **cosine rule** can be used to **find missing angles** and **sides** in **non-right angled triangles**.

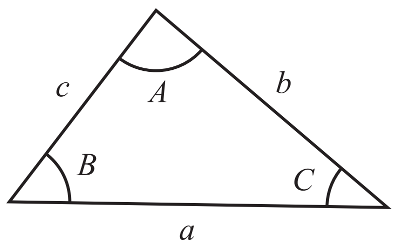
Use the cosine rule if you **do not** have **an angle** and **the side opposite** it.

The **cosine rule** is given in the formula book and is as follows:

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

(Only one of form is given in the formula book – swap letters for others!).

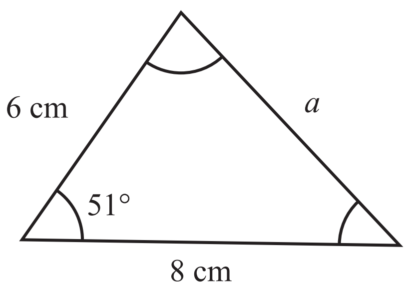
The triangle is labelled up in the usual way:



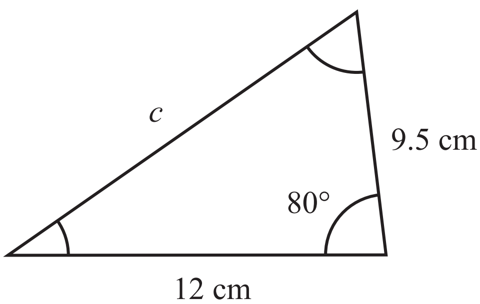
Note: other letters may be used, the structure of the formula is the same.

Use the **cosine rule** to find all the **missing side** in these triangles:

(1)



(2)



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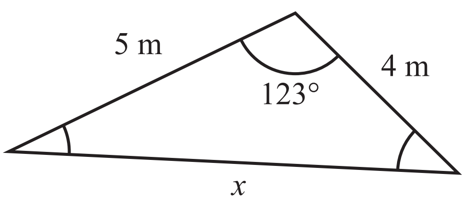

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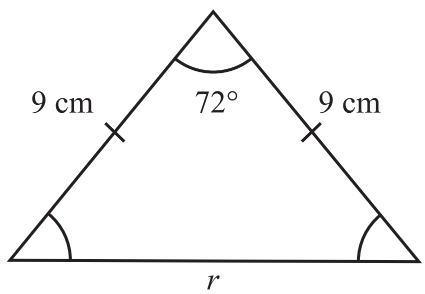
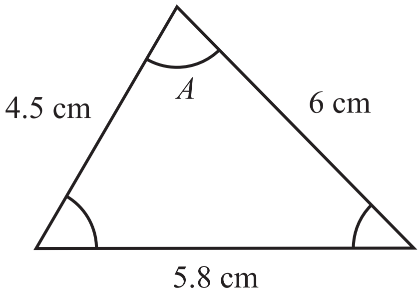


Trigonometry (Non-Right Angled Triangles) - The Cosine Rule

<u>Exercise B:</u> The Cosine Rule

Use the **cosine rule** to find the labeled **missing angle** in these triangles.

(5)



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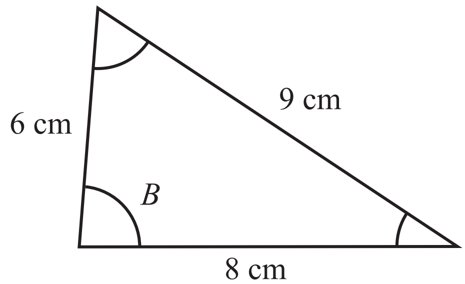

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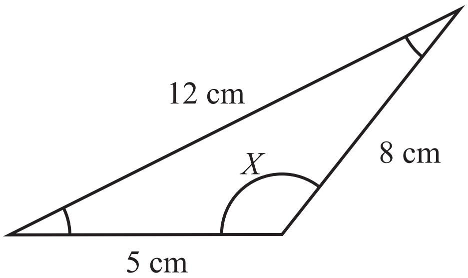

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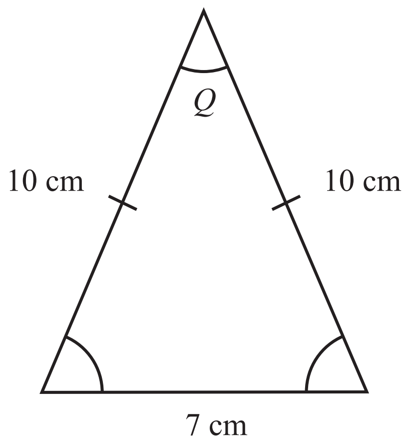
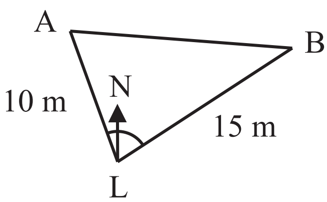


(9) Teams A and B start from a lodge L.

Team A walks 10 miles on a bearing of 340° and team B walks 15 miles on a bearing of 057°.

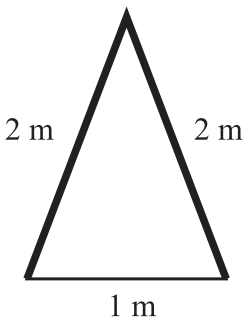


(a) explain why the marked angle is 77°?

(b) calculate the distance between the teams after the walk

(c) what is the bearing of team A from Team B?

(10) A step ladder makes an isosceles triangle as shown:



The length of the ladder is 2 m.

The feet are placed 1 m apart.

(a) use the cosine rule to calculate the angle at the apex *A*

(b) now split the triangle into two right-angled triangles and calculate the angle this way to verify you get the same result!

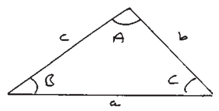
P. Leary

Trigonometry (Non-Right Angled Triangles) - The Area Formula

TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

THE AREA FORMULA

The area of this triangle



can be found using the formula :

$$\boxed{A = \frac{1}{2}ab\sin C}$$ 'included angle'

this also yields two other symmetric formulae

$$\boxed{\begin{aligned} A &= \frac{1}{2}ac\sin B \\ A &= \frac{1}{2}bc\sin A \end{aligned}}$$

Note if the angles had been marked P, Q and R with sides p, q and r the following formulae could be written

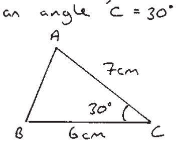
$A = \frac{1}{2}pq\sin R$ $A = \frac{1}{2}pr\sin Q$ $A = \frac{1}{2}qr\sin P$

TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

THE AREA FORMULA

EXAMPLE 1

A triangle ABC has sides BC = 6cm, AC = 7cm and an angle C = 30°.



Calculate the area of this triangle.

$a = 6$
$b = 7$
$C = 30°$

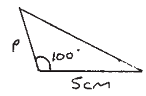
$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2} \times 6 \times 7 \times \sin 30°$

$A = 10.5\text{ cm}^2$

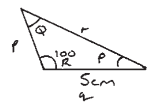
EXAMPLE 2

This triangle has an area of 9.85 cm².



Find the length of the side labelled p.

Start by labelling up sides



Required formula

$p = ?$
$q = 5$
$R = 100°$

$A = \frac{1}{2}pq\sin R$

substituting

$9.85 = \frac{1}{2}p \times 5 \times \sin 100$

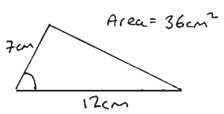
$p = \frac{9.85 \times 2}{5 \times \sin 100} = 4.0\text{cm}$

TRIGONOMETRY NON-RIGHT ANGLED TRIANGLES

THE AREA FORMULA

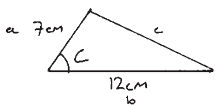
EXAMPLE 3

Given this triangle



Calculate the size of the included angle

Start by labelling up the sides



Required formula

$A = \frac{1}{2}ab\sin C$

substituting values

$36 = \frac{1}{2} \times 7 \times 12 \times \sin C$

$\sin C = \frac{36 \times 2}{7 \times 12}$

$C = \sin^{-1}\left(\frac{36 \times 2}{7 \times 12}\right)$

$C = 59°$

Trigonometry (Non-Right Angled Triangles) - The Area Formula

Exercise C: The Area Formula

The **area** of any **triangle** can be found using the **area formula**:

$$\text{Area} = \tfrac{1}{2}ab\sin C$$

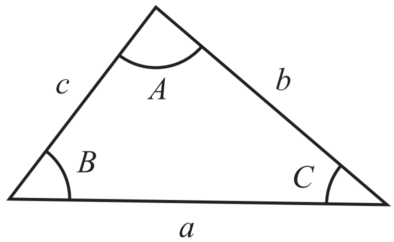
$$\text{Area} = \tfrac{1}{2}bc\sin A$$

$$\text{Area} = \tfrac{1}{2}ac\sin B$$

You need **two sides** and **the included angle**.

Change between the forms by swapping letters.

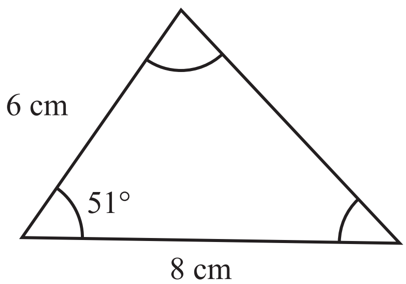
The triangle is labelled up in the usual way:



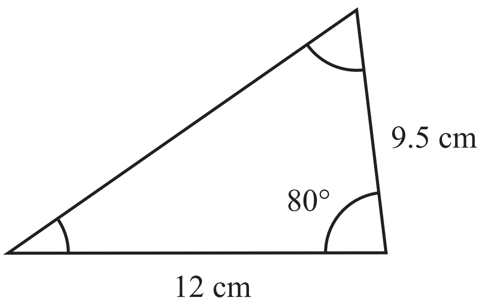
Note: other letters may be used, the structure of the formula is the same.

Calculate the **areas** in the following triangles:

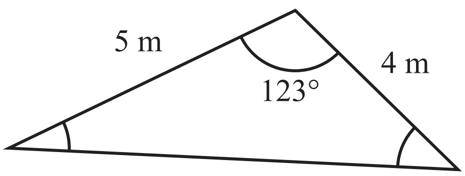
(1)



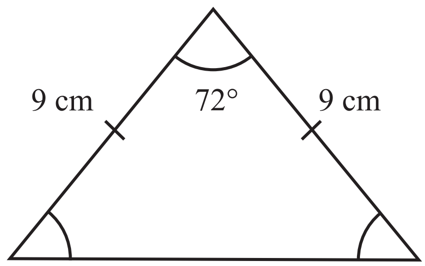
(2)



(3)



(4)

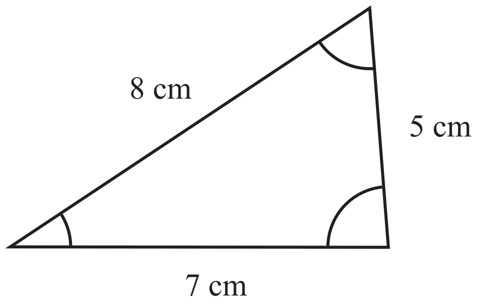


Trigonometry (Non-Right Angled Triangles) - The Area Formula

Exercise C: The Area Formula

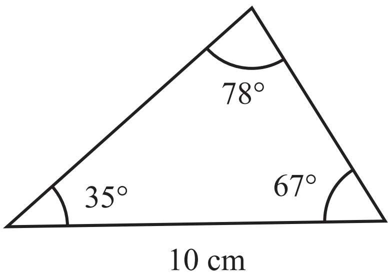
Find the **area** of this triangle (you will need to **find** an **angle** first!).

(5)



Find the **area** of this triangle (you will need to **find** another **side** first!).

(6)



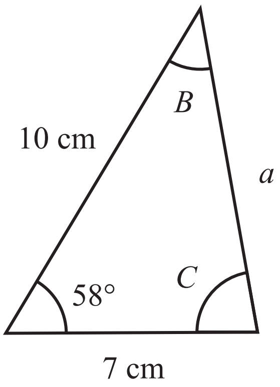
Trigonometry (Non-Right Angled Triangles) - Mixed Problems

<u>Exercise D:</u> Mixed Problems

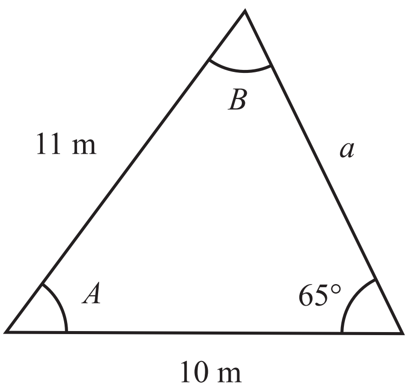
Solving a **triangle** means **finding all missing sides** and **angles**.

Solve the **triangles** in questions 1-3.

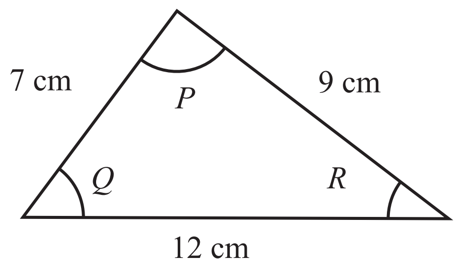
(1)



(2)



(3)



THE SINE RULE

EXERCISE A

① $\frac{\sin C}{c} = \frac{\sin A}{A}$

$\frac{\sin C}{5} = \frac{\sin 75}{8}$

$\sin C = \frac{5\sin 75}{8} = 0.6037....$

$C = \sin^{-1}(0.6037....)$

$C = 37.1°$

$B = 180° - 75° - 37.1° = 67.9°$

② $\frac{\sin X}{x} = \frac{\sin Z}{z}$

$\frac{\sin X}{7.5} = \frac{\sin 112}{12}$

$\sin X = \frac{7.5\sin 112}{12} = 0.5794....$

$X = \sin^{-1}(0.5794...)$

$X = 35.4°$

$Y = 180° - 112° - 35.4° = 32.6°$

③ $\frac{\sin P}{p} = \frac{\sin R}{r}$

$\frac{\sin P}{9.2} = \frac{\sin 50}{9.6}$

$\sin P = \frac{9.2\sin 50}{9.6} = 0.7341...$

$P = \sin^{-1}(0.7341....)$

$P = 47.2°$

$Q = 180 - 50 - 47.2 = 82°$

④ $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{2} = \frac{\sin 94}{5}$

$\sin A = \frac{2\sin 94}{5} = 0.399...$

$A = \sin^{-1}(0.399....)$

$A = 23.5°$

$B = 180 - 94 - 23.5 = 62.5°$

⑤ $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 80} = \frac{10}{\sin 50}$

$a = \frac{10\sin 80}{\sin 50}$

$a = 12.9$ cm

⑥ $\frac{x}{\sin X} = \frac{y}{\sin Y}$

$\frac{x}{\sin 40} = \frac{7}{\sin 75}$

$x = \frac{7\sin 40}{\sin 75}$

$x = 4.7$cm

THE SINE RULE

EXERCISE 1A

(7)
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 18^\circ} = \frac{15}{\sin 110}$$

$$c = \frac{15 \sin 18}{\sin 110}$$

$$c = 4.9\text{ cm}$$

(8)
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 24} = \frac{12}{\sin 132}$$

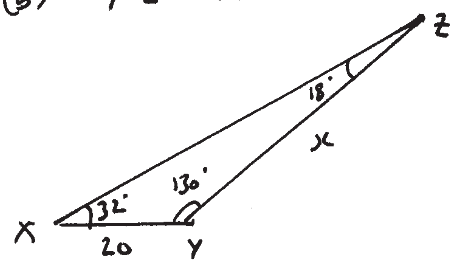
$$b = \frac{12 \sin 24}{\sin 132}$$

$$b = 6.6\text{ m}$$

(9) (a) $XYZ = 180 - 50 = 130^\circ$

$XZY = 180 - 130 - 32 = 18^\circ$

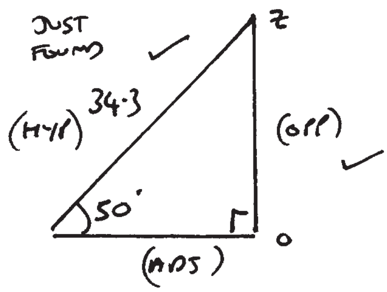
(b) $YZ = x$



$$\frac{x}{\sin 32} = \frac{20}{\sin 18}$$

$$x = \frac{20 \sin 32}{\sin 18} = 34.3\text{ m}$$

(c)



$$\sin 50 = \frac{\text{opp}}{34.3}$$

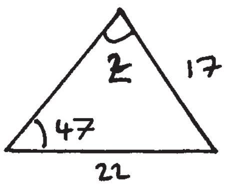
$$\text{opp} = 34.3 \sin 50$$

$$\text{opp} = 26.3\text{ m}$$

(10) (a) $90 - 43 = 47^\circ$,

Interior angles (sum $= 180^\circ$)
$180 - 43 = 137^\circ$

(b)



$$\frac{\sin Z}{22} = \frac{\sin 43}{17}$$

$$\sin Z = \frac{22 \sin 47}{17} = 0.946\ldots$$

$$Z = \sin^{-1}(0.946\ldots) = 71.2^\circ$$

(c) $y + z = 137 + 71 = 208^\circ$

remaining angle is bearing $360 - 208 = 152^\circ$

THE COSINE RULE — EXERCISE B

(1) $a^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 51$

$a^2 = 39.6$

$a = \sqrt{39.6} = 6.3\text{ cm}$

(2) $c^2 = 12^2 + 9.5^2 - 2 \times 12 \times 9.5 \cos 80$

$c^2 = 194.7$

$c = \sqrt{194.7} = 14.0\text{ cm}$

(3) $x^2 = 5^2 + 4^2 - 2 \times 5 \times 4 \cos 123$

$x^2 = 62.8$

$x = \sqrt{62.8} = 7.9\text{ cm}$

(4) $r^2 = 9^2 + 9^2 - 2 \times 9 \times 9 \cos 72$

$r^2 = 111.9$

$r = \sqrt{111.9} = 10.6\text{ cm}$

(5) $a^2 = b^2 + c^2 - 2bc\cos A$

$5.8^2 = 4.5^2 + 6^2 - 2 \times 4.5 \times 6 \cos A$

$33.64 = 56.25 - 54\cos A$

$54\cos A = 22.61$

$\cos A = \dfrac{22.61}{54}$

$A = \cos^{-1}\left(\dfrac{22.61}{54}\right) = 65°$

(6) $b^2 = a^2 + c^2 - 2ac\cos B$

$9^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos B$

$81 = 100 - 96\cos B$

$96\cos B = 19$

$\cos B = \dfrac{19}{96}$

$B = \cos^{-1}\left(\dfrac{19}{96}\right) = 79°$

(7) $x^2 = y^2 + z^2 - 2yz\cos X$

$12^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \cos X$

$144 = 89 - 80\cos X$

$80\cos X = -55$

$\cos X = \dfrac{-55}{80}$

$X = \cos^{-1}\left(\dfrac{-55}{80}\right) = 133°$

(8) $q^2 = p^2 + r^2 - 2pr\cos Q$

$7^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \cos Q$

$49 = 200 - 200\cos Q$

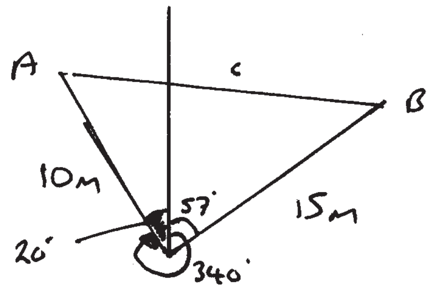
$200\cos Q = 151$

$\cos Q = \dfrac{151}{200}$

$Q = \cos^{-1}\left(\dfrac{151}{200}\right) = 41°$

THE COSINE RULE — EXERCISE B

(9)



(a) $20 + 57 = 77°$

(b) $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \cos 77$

$c^2 = 257.5$

$c = 16$ m

Find angle B (other ways possible)

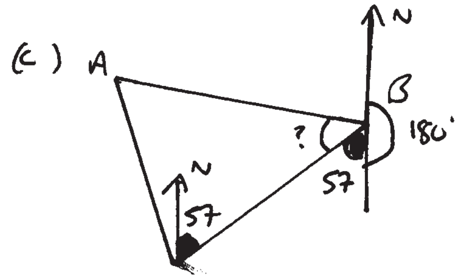
$$\frac{\sin B}{10} = \frac{\sin 77}{16}$$

$$\sin B = \frac{10 \sin 77}{16}$$

$\sin B = 0.608\ldots$

$B = \sin^{-1}(0.608\ldots) = 38°$

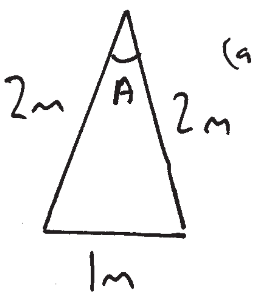
(c)



$\therefore$ Bearing A from B =

$180 + 57 + 38 = 275°$

(10)



(a) $a^2 = b^2 + c^2 - 2bc\cos A$

$1^2 = 2^2 + 2^2 - 2 \times 2 \times 2 \cos A$

$1 = 8 - 8\cos A$

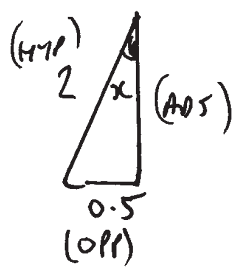
$8\cos A = 7$

$\cos A = \frac{7}{8}$

$A = \cos^{-1}\left(\frac{7}{8}\right)$

$A = 29°$.

(b)



$\sin x = \frac{0.5}{2}$

$\sin x = 0.25$

$x = \sin^{-1}(0.25) = 14.5°$

apex $= 2x = 2 \times 14.5 = 29°$

AREA — EXERCISE C

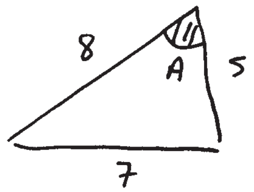
1. $\text{Area} = \frac{1}{2} \times 6 \times 8 \times \sin 51 = 18.7 \text{ cm}^2$

2. $\text{Area} = \frac{1}{2} \times 12 \times 9.5 \times \sin 80 = 56.1 \text{ cm}^2$

3. $\text{Area} = \frac{1}{2} \times 5 \times 4 \times \sin 123 = 8.4 \text{ m}^2$

4. $\text{Area} = \frac{1}{2} \times 9 \times 9 \times \sin 72 = 38.5 \text{ cm}^2$

5.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$7^2 = 5^2 + 8^2 - 2 \times 5 \times 8\cos A$$

$$49 = 89 - 80\cos A$$

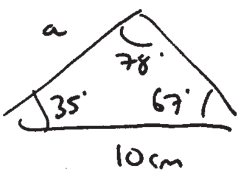
$$80\cos A = 40$$

$$\cos A = \frac{40}{80}$$

$$A = \cos^{-1}\left(\frac{40}{80}\right) = 60^\circ$$

$\therefore \text{Area} = \frac{1}{2} \times 5 \times 8 \times \sin 60 = 17.3 \text{ cm}^2$

6.



$$\frac{a}{\sin 67} = \frac{10}{\sin 78}$$

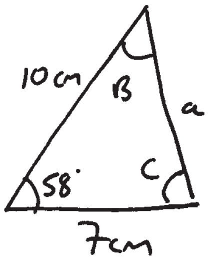
$$a = \frac{10\sin 67}{\sin 78} = 9.4 \text{ cm}$$

$\text{Area} = \frac{1}{2} \times 9.4 \times 10 \times \sin 35 = 27.0 \text{ cm}^2$

OTHER WAYS!!

MIXED EXERCISE D

(1)



$a^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 58$

$a^2 = 74.8 \rightarrow a = 8.65\text{ cm}$

$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin B}{7} = \frac{\sin 58}{8.65}$ $\quad \sin B = \frac{7 \sin 58}{8.65} = 0.686\ldots$

$B = \sin^{-1}(0.686\ldots) = 43°$

$C = 180 - 58 - 43 = 79°$

(2) $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin B}{10} = \frac{\sin 65}{11}$

$\sin B = \frac{10 \sin 65}{11} = 0.82\ldots$

$B = \sin^{-1}(0.82\ldots) = 55°$

$A = \cancel{65+} 180 - 65 - 55 = 60°$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 60} = \frac{11}{\sin 65}$

$a = \frac{11 \sin 60}{\sin 65}$ $\quad a = 10.5\text{ cm}$

(3) $q^2 = p^2 + r^2 - 2pr \cos Q$

$q^2 = 7^2 + 12^2 - 2 \times 7 \times 12 \cos Q$

$81 = 193 - 168 \cos Q$

$168 \cos Q = 112$

$\cos Q = \frac{112}{168}$

$Q = \cos^{-1}\left(\frac{112}{168}\right)$ $\quad Q = 48°$

$p^2 = r^2 + q^2 - 2rq \cos P$

$12^2 = 7^2 + 9^2 - 2 \times 7 \times 9 \cos P$

$144 = 130 - 126 \cos P$

$126 \cos P = -14$

$\cos P = \frac{-14}{126}$

$P = \cos^{-1}\left(\frac{-14}{126}\right)$ $\quad P = 96°$

$R = 180 - 48 - 96$ $\quad R = 36°$