

Completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	1 Write $x^2 + bx + c$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ 2 Simplify
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Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ 2 Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ 3 Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2 4 Simplify
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Practice

- 1 Write the following quadratic expressions in the form $(x + p)^2 + q$
- | | |
|-------------------------|--------------------------|
| a $x^2 + 4x + 3$ | b $x^2 - 10x - 3$ |
| c $x^2 - 8x$ | d $x^2 + 6x$ |
| e $x^2 - 2x + 7$ | f $x^2 + 3x - 2$ |
- 2 Write the following quadratic expressions in the form $p(x + q)^2 + r$
- | | |
|---------------------------|---------------------------|
| a $2x^2 - 8x - 16$ | b $4x^2 - 8x - 16$ |
| c $3x^2 + 12x - 9$ | d $2x^2 + 6x - 8$ |
- 3 Complete the square.
- | | |
|--------------------------|--------------------------|
| a $2x^2 + 3x + 6$ | b $3x^2 - 2x$ |
| c $5x^2 + 3x$ | d $3x^2 + 5x + 3$ |

Extend

- 4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

1 a $(x + 2)^2 - 1$

b $(x - 5)^2 - 28$

c $(x - 4)^2 - 16$

d $(x + 3)^2 - 9$

e $(x - 1)^2 + 6$

f $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

2 a $2(x - 2)^2 - 24$

b $4(x - 1)^2 - 20$

c $3(x + 2)^2 - 21$

d $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

3 a $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$

b $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

c $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$

d $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

4 $(5x + 3)^2 + 3$