

# Straight line graphs

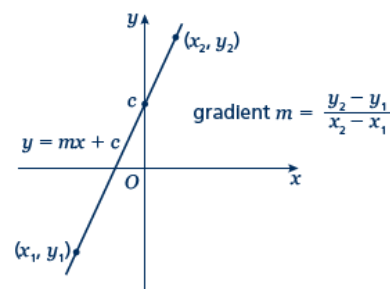
## A LEVEL LINKS

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

## Key points

- A straight line has the equation  $y = mx + c$ , where  $m$  is the gradient and  $c$  is the y-intercept (where  $x = 0$ ).
- The equation of a straight line can be written in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.
- When given the coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$  of two points on a line the gradient is calculated using the

$$\text{formula } m = \frac{y_2 - y_1}{x_2 - x_1}$$



## Examples

**Example 1** A straight line has gradient  $-\frac{1}{2}$  and y-intercept 3.

Write the equation of the line in the form  $ax + by + c = 0$ .

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	<ol style="list-style-type: none"> <li>1 A straight line has equation <math>y = mx + c</math>. Substitute the gradient and y-intercept given in the question into this equation.</li> <li>2 Rearrange the equation so all the terms are on one side and 0 is on the other side.</li> <li>3 Multiply both sides by 2 to eliminate the denominator.</li> </ol>
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**Example 2** Find the gradient and the y-intercept of the line with the equation  $3y - 2x + 4 = 0$ .

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient $= m = \frac{2}{3}$ y-intercept $= c = -\frac{4}{3}$	<ol style="list-style-type: none"> <li>1 Make <math>y</math> the subject of the equation.</li> <li>2 Divide all the terms by three to get the equation in the form <math>y = \dots</math></li> <li>3 In the form <math>y = mx + c</math>, the gradient is <math>m</math> and the y-intercept is <math>c</math>.</li> </ol>
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**Example 3** Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$  $13 = 3 \times 5 + c$  $13 = 15 + c$ $c = -2$ $y = 3x - 2$	<ol style="list-style-type: none"> <li>1 Substitute the gradient given in the question into the equation of a straight line <math>y = mx + c</math>.</li> <li>2 Substitute the coordinates <math>x = 5</math> and <math>y = 13</math> into the equation.</li> <li>3 Simplify and solve the equation.</li> <li>4 Substitute <math>c = -2</math> into the equation <math>y = 3x + c</math></li> </ol>
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**Example 4** Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2, x_2 = 8, y_1 = 4 \text{ and } y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$  $y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ $c = 3$ $y = \frac{1}{2}x + 3$	<ol style="list-style-type: none"> <li>1 Substitute the coordinates into the equation <math>m = \frac{y_2 - y_1}{x_2 - x_1}</math> to work out the gradient of the line.</li> <li>2 Substitute the gradient into the equation of a straight line <math>y = mx + c</math>.</li> <li>3 Substitute the coordinates of either point into the equation.</li> <li>4 Simplify and solve the equation.</li> <li>5 Substitute <math>c = 3</math> into the equation <math>y = \frac{1}{2}x + c</math></li> </ol>
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## Practice

**1** Find the gradient and the y-intercept of the following equations.

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|----------------------------|----------------------------------|
| <b>a</b> $y = 3x + 5$      | <b>b</b> $y = -\frac{1}{2}x - 7$ |
| <b>c</b> $2y = 4x - 3$     | <b>d</b> $x + y = 5$             |
| <b>e</b> $2x - 3y - 7 = 0$ | <b>f</b> $5x + y - 4 = 0$        |

**Hint**  
Rearrange the equations to the form  $y = mx + c$

**2** Copy and complete the table, giving the equation of the line in the form  $y = mx + c$ .

Gradient	y-intercept	Equation of the line
5	0	
-3	2	
4	-7	

- 3** Find, in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are integers, an equation for each of the lines with the following gradients and y-intercepts.
- a** gradient  $-\frac{1}{2}$ , y-intercept  $-7$       **b** gradient  $2$ , y-intercept  $0$
- c** gradient  $\frac{2}{3}$ , y-intercept  $4$       **d** gradient  $-1.2$ , y-intercept  $-2$
- 4** Write an equation for the line which passes through the point  $(2, 5)$  and has gradient  $4$ .
- 5** Write an equation for the line which passes through the point  $(6, 3)$  and has gradient  $-\frac{2}{3}$ .
- 6** Write an equation for the line passing through each of the following pairs of points.
- a**  $(4, 5)$ ,  $(10, 17)$       **b**  $(0, 6)$ ,  $(-4, 8)$
- c**  $(-1, -7)$ ,  $(5, 23)$       **d**  $(3, 10)$ ,  $(4, 7)$

## Extend

- 7** The equation of a line is  $2y + 3x - 6 = 0$ .  
Write as much information as possible about this line.

## Answers

- 1   **a**    $m = 3, c = 5$                                       **b**    $m = -\frac{1}{2}, c = -7$   
      **c**    $m = 2, c = -\frac{3}{2}$                                       **d**    $m = -1, c = 5$   
      **e**    $m = \frac{2}{3}, c = -\frac{7}{3}$  or  $-2\frac{1}{3}$                                       **f**    $m = -5, c = 4$

2

Gradient	y-intercept	Equation of the line
5	0	$y = 5x$
-3	2	$y = -3x + 2$
4	-7	$y = 4x - 7$

- 3   **a**    $x + 2y + 14 = 0$                                       **b**    $2x - y = 0$   
      **c**    $2x - 3y + 12 = 0$                                       **d**    $6x + 5y + 10 = 0$

4    $y = 4x - 3$

5    $y = -\frac{2}{3}x + 7$

6   **a**    $y = 2x - 3$                                       **b**    $y = -\frac{1}{2}x + 6$

**c**    $y = 5x - 2$                                       **d**    $y = -3x + 19$

7    $y = -\frac{3}{2}x + 3$ , the gradient is  $-\frac{3}{2}$  and the y-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as  $\left(1, \frac{3}{2}\right)$  or  $(4, -3)$ .