Rearranging equations

A LEVEL LINKS
Scheme of work: 6a. Definition, differentiating polynomials, second derivatives
Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$ $v - u = at$ $t = \frac{v-u}{a}$	**1** Get the terms containing t on one side and everything else on the other side. **2** Divide throughout by a.

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$ $r = t(2 - \pi)$ $t = \frac{r}{2-\pi}$	**1** All the terms containing t are already on one side and everything else is on the other side. **2** Factorise as t is a common factor. **3** Divide throughout by $2 - \pi$.

Example 3 Make t the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$ $2t + 2r = 15t$ $2r = 13t$ $t = \frac{2r}{13}$	**1** Remove the fractions first by multiplying throughout by 10. **2** Get the terms containing t on one side and everything else on the other side and simplify. **3** Divide throughout by 13.

Example 4 Make t the subject of the formula $r = \dfrac{3t+5}{t-1}$.

$r = \dfrac{3t+5}{t-1}$ $r(t-1) = 3t+5$ $rt - r = 3t + 5$ $rt - 3t = 5 + r$ $t(r-3) = 5 + r$ $t = \dfrac{5+r}{r-3}$	**1** Remove the fraction first by multiplying throughout by $t - 1$. **2** Expand the brackets. **3** Get the terms containing t on one side and everything else on the other side. **4** Factorise the LHS as t is a common factor. **5** Divide throughout by $r - 3$.

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ $[d]$ **2** $P = 2l + 2w$ $[w]$ **3** $D = \dfrac{S}{T}$ $[T]$

4 $p = \dfrac{q-r}{t}$ $[t]$ **5** $u = at - \dfrac{1}{2}t$ $[t]$ **6** $V = ax + 4x$ $[x]$

7 $\dfrac{y-7x}{2} = \dfrac{7-2y}{3}$ $[y]$ **8** $x = \dfrac{2a-1}{3-a}$ $[a]$ **9** $x = \dfrac{b-c}{d}$ $[d]$

10 $h = \dfrac{7g-9}{2+g}$ $[g]$ **11** $e(9 + x) = 2e + 1$ $[e]$ **12** $y = \dfrac{2x+3}{4-x}$ $[x]$

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \dfrac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \dfrac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\dfrac{xy}{z} = \dfrac{ab}{cd}$ **b** $\dfrac{4\pi cx}{d} = \dfrac{3z}{py^2}$

15 Make $\sin B$ the subject of the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac\cos B$.

Extend

17 Make x the subject of the following equations.

a $\dfrac{p}{q}(sx+t) = x-1$ **b** $\dfrac{p}{q}(ax+2y) = \dfrac{3p}{q^2}(x-y)$

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Answers

1 $d = \frac{C}{\pi}$ **2** $w = \frac{P - 2l}{2}$ **3** $T = \frac{S}{D}$

4 $t = \frac{q - r}{p}$ **5** $t = \frac{2u}{2a - 1}$ **6** $x = \frac{V}{a + 4}$

7 $y = 2 + 3x$ **8** $a = \frac{3x + 1}{x + 2}$ **9** $d = \frac{b - c}{x}$

10 $g = \frac{2h + 9}{7 - h}$ **11** $e = \frac{1}{x + 7}$ **12** $x = \frac{4y - 3}{2 + y}$

13 **a** $r = \sqrt{\frac{A}{\pi}}$ **b** $r = \sqrt[3]{\frac{3V}{4\pi}}$

c $r = \frac{P}{\pi + 2}$ **d** $r = \sqrt{\frac{3V}{2\pi h}}$

14 **a** $x = \frac{abz}{cdy}$ **b** $x = \frac{3dz}{4\pi cpy^2}$

15 $\sin B = \frac{b \sin A}{a}$

16 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

17 **a** $x = \frac{q + pt}{q - ps}$ **b** $x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3 + 2q)}{3 - aq}$

Pearson