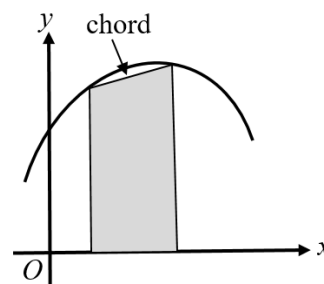# Area under a graph

> **A LEVEL LINKS**
> **Scheme of work:** 7b. Definite integrals and areas under curves

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.
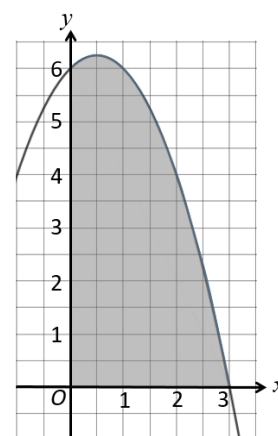

- The area of a trapezium $= \dfrac{1}{2}h(a+b)$
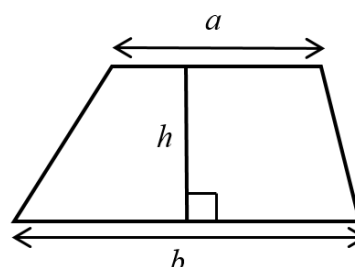
Examples

Example 1   Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the x-axis from $x = 0$ to $x = 3$. Use three strips of width 1 unit.

x	0	1	2	3
$y = (3 - x)(2 + x)$	6	6	4	0

Trapezium 1:
$a_1 = 6 - 0 = 6$, $b_1 = 6 - 0 = 6$
Trapezium 2:
$a_2 = 6 - 0 = 6$, $b_2 = 4 - 0 = 4$
Trapezium 3:
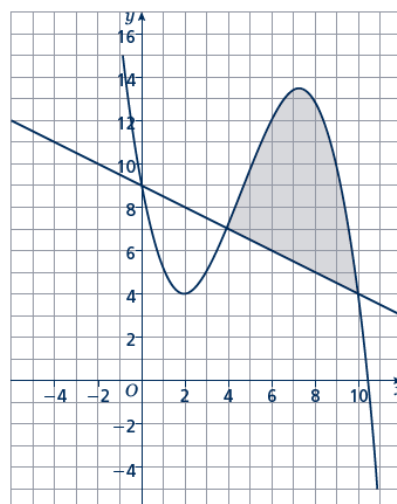$a_3 = 4 - 0 = 4$, $a_3 = 0 - 0 = 0$

1 Use a table to record the value of y on the curve for each value of x.

2 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the x-axis give the values for a.

(continued on next page)

$$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 1(6+6) = 6$$

$$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 1(6+4) = 5$$

$$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 1(4+0) = 2$$

Area = 6 + 5 + 2 = 13 units^2

3 Work out the area of each trapezium. $h = 1$ since the width of each trapezium is 1 unit.

4 Work out the total area. Remember to give units with your answer.

Example 2 Estimate the shaded area.
Use three strips of width 2 units.

x	4	6	8	10
y	7	12	13	4

1 Use a table to record y on the curve for each value of x.

x	4	6	8	10
y	7	6	5	4

2 Use a table to record y on the straight line for each value of x.

Trapezium 1:
$a_1 = 7 - 7 = 0$, $b_1 = 12 - 6 = 6$
Trapezium 2:
$a_2 = 12 - 6 = 6$, $b_2 = 13 - 5 = 8$
Trapezium 3:
$a_3 = 13 - 5 = 8$, $a_3 = 4 - 4 = 0$

3 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.

$$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0+6) = 6$$

$$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6+8) = 14$$

$$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8+0) = 8$$

Area = 6 + 14 + 8 = 28 units^2

4 Work out the area of each trapezium. $h = 2$ since the width of each trapezium is 2 units.

5 Work out the total area. Remember to give units with your answer.
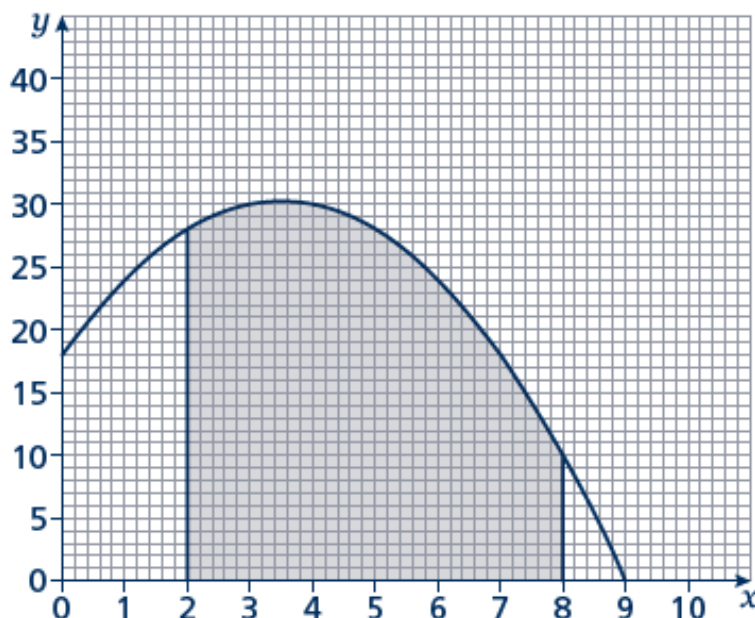
Practice

1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
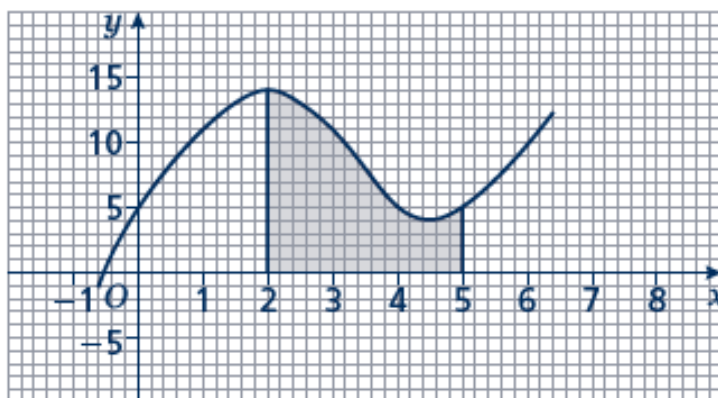Use four strips of width 1 unit.

2 Estimate the shaded area shown on the axes.
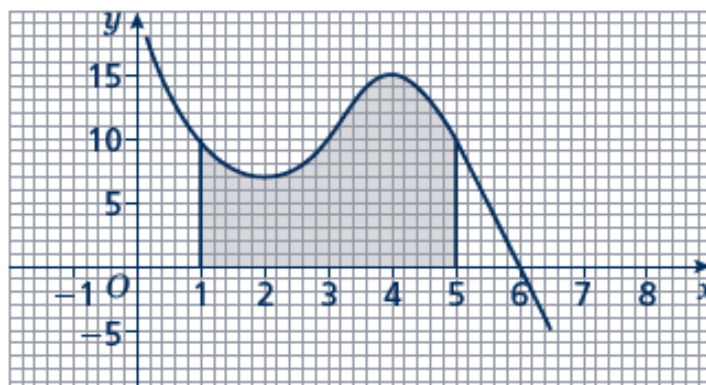Use six strips of width 1 unit.

3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.

5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the
x-axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

6 Estimate the shaded area.
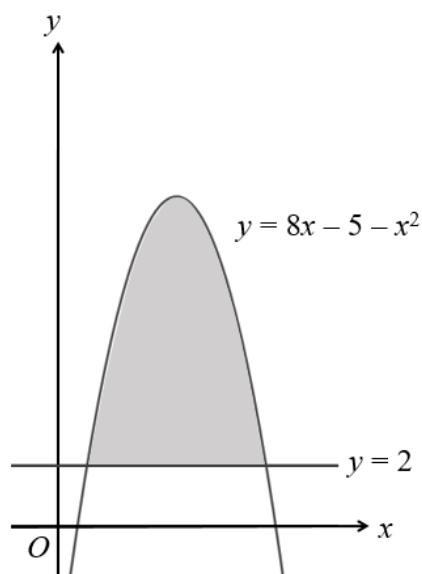Use four strips of equal width.

7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the
x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

8 Estimate the shaded area.
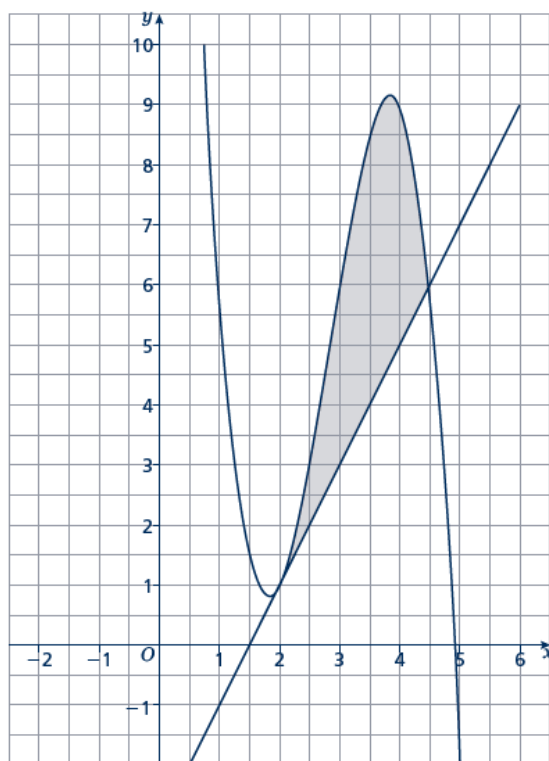Use seven strips of equal width.
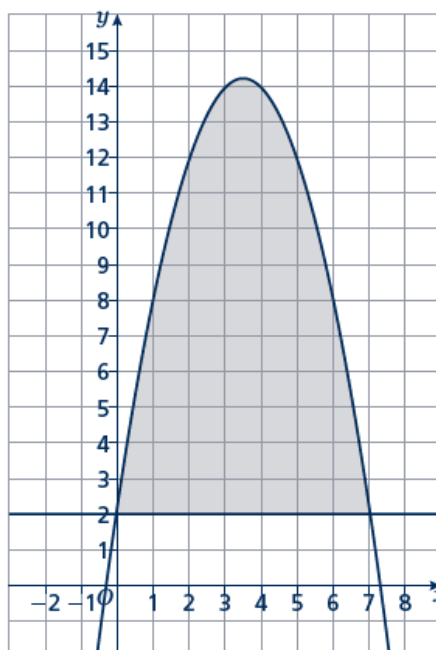
Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$
are shown in the sketch.
Estimate the shaded area using six strips
of equal width.

$y = 8x - 5 - x^2$

$y = 2$

10 Estimate the shaded area using five strips of equal width.

Answers

1   34 units^2

2   149 units^2

3   14 units^2

4 $25\frac{1}{4}$ units^2

5   35 units^2

6   42 units^2

7 $26\frac{7}{8}$ units^2

8   56 units^2

9   35 units^2

10 $6\frac{1}{4}$ units^2