

What do I need to be able to do?

By the end of this chapter you should be able to:

- Multiply and divide integer powers
- Expand a single term of brackets and collect like terms
- Expand the product of two or three expressions
- Factorise linear, quadratic and simple cubic expressions
- Know and use the laws of indices
- Simplify and use the rules of surds
- Rationalise denominators

Y12 – Chapter 1 Algebraic Expressions

Key words:

- Integer – A number with no fractional part (no decimals)
- Product – The answer when two or more values are multiplied together
- Surd – A number that can't be simplified to remove a square root (or cube root etc)
- Irrational – A real number that can NOT be made by dividing two integers eg π
- Rational – A number that can be made by dividing two integers
- Base – The number that gets multiplied when using an exponent (index/power)

Expanding and factorising

Expanding and factorising are the inverse of each other

$$\begin{array}{c}
 \text{Expanding brackets} \\
 4x(2x + y) = 8x^2 + 4xy \\
 (x + 5)^3 = x^3 + 15x^2 + 75x + 125 \\
 (x + 2y)(x - 5y) = x^2 - 3xy - 10y^2 \\
 \text{Factorising}
 \end{array}$$

Pure Maths Year 1/AS

Indices

An index (power) tells you how many times to multiply something by itself:
eg x^5 means $x \times x \times x \times x \times x$

There is a base and a power eg:

$$\text{base} \rightarrow a^m \leftarrow \text{power}$$

Surds

Writing surds in their simplest form

If a square root has a perfect square number as a factor, then it can be simplified.
eg $\sqrt{20}$ can be re-written as $\sqrt{4 \times 5}$ which simplifies to $2\sqrt{5}$

Perfect square

Adding and subtracting surds

Remember to add or subtract like terms (ie. the rational numbers and the roots (of the same number))

$$\text{eg } (7+3\sqrt{2})+(8-\sqrt{2})=15+2\sqrt{2} \quad \begin{array}{l} \text{Add rational parts: } (7+8=15) \\ \text{Add roots: } (3\sqrt{2}-1\sqrt{2}=2\sqrt{2}) \end{array}$$

Multiplying surds

If there is no rational part then multiplying is easy: eg $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

If there is a rational part then multiply out the brackets

$$\text{eg } (5+\sqrt{3})(2-\sqrt{3}) = 10 - 5\sqrt{3} + 2\sqrt{3} - \sqrt{3}\sqrt{3} \text{ ties up to give } 7 - 3\sqrt{3}$$

Rationalising the denominator

You rationalise the denominator to get rid of the surd on the bottom of a fraction.
To rationalise the denominator just multiply the top and bottom of the fraction by the bottom of the fraction with the opposite sign in front of the root.

$$\text{eg } \frac{3+\sqrt{5}}{2-\sqrt{5}} \quad \text{We are just finding an equivalent fraction by multiplying by 1 (just in disguise)}$$

$$\frac{3+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{6+3\sqrt{5}+2\sqrt{5}+\sqrt{5}\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-\sqrt{5}\sqrt{5}} = \frac{11+5\sqrt{5}}{-1} = -11 - 5\sqrt{5}$$

Notice these are the same – but the sign in front of the root has changed

Changing the sign in front of the root makes the middle parts cancel each other out

| Rule | Meaning |
|-------------------------------------|---|
| $a^m \times a^n = a^{m+n}$ | To multiply 2 numbers with the same base you add the powers. |
| $\frac{a^m}{a^n} = a^{m-n}$ | To divide 2 numbers with the same base you subtract the powers. |
| $(a^m)^n = a^{mn}$ | To simplify a power inside and outside of a bracket you multiply the powers. |
| $a^{-m} = \frac{1}{a^m}$ | A negative power means find the reciprocal ("one over") so send everything to the bottom of a fraction. |
| $\frac{a^m}{a^n} = (\sqrt[n]{a})^m$ | A fractional power means a root. Denominator tells you the root and the numerator tells you the power. |
| $a^0 = 1$ | Anything to the power of zero = 1 |
| $a^1 = a$ | Any number to the power of one stays the same |



Pure Mathematics PLC Checklist

| Chapter 1 | | Algebraic Expressions | | | | | | | | | | | | | | | | | | | | RAG | | | | | | | |
|------------------|--|--|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|----|--|--|--|--|
| 1.1 | | Use the laws of indices to simplify expressions. | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exercise 1A | | 1 | 2 | 3 | | | | | | | | | | | | | | | | | | EX | HW | ASS | | | | | |
| 1.2 | | Expand brackets and simplify expressions (double and triple brackets). | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exercise 1B | | 1 | 2 | 3 | 4 | 5 | | | | | | | | | | | | | | | | EX | HW | ASS | | | | | |
| 1.3 | | Factorise expressions into a single bracket, quadratics and simple cubics. | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exercise 1C | | 1 | 2 | 3 | 4 | 5 | | | | | | | | | | | | | | | | EX | HW | ASS | | | | | |
| 1.4 | | Work with fractional and negative indices. | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exercise 1D | | 1 | 2 | 3 | 4 | 5 | | | | | | | | | | | | | | | | EX | HW | ASS | | | | | |
| 1.5 | | Simplify expressions involving surds including multiplying brackets. | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exercise 1E | | 1 | 2 | 3 | | | | | | | | | | | | | | | | | | EX | HW | ASS | | | | | |
| 1.6 | | Rationalise the denominator of a surd. | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Exercise 1F | | 1 | 2 | 3 | 3 | | | | | | | | | | | | | | | | EX | HW | ASS | | | | | | |
| Mixed Exercise 1 | | Review questions from chapter (some may be set as part of homework). | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | | | | |

Independent Learning and Revision Opportunities

You should independently work on these questions over the year as ongoing revision.

Revision questions from the book covering the new specification.

| | | | | | | | | | | | | | | | | | | | | | | | | |
|-----------|----------------------------------|---|---|---|---|---|---|---|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| Book | Review Exercise 1 (Pages 85-88). | | | | | | | | | | | | | | | | | | | | | | | |
| Questions | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | | | | | | | | | | | | | | | |

Examination questions on the topic from the old specification (paper pack on Firefly).

<https://chellaston.fireflycloud.net/maths-1/year-12/as-maths-revision/as-pure-revision-papers>

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---------------|--|---|---|---|---|---|---|---|---|----|----|----|--|--|--|--|--|--|--|--|--|--|--|--|
| Revision Pack | AS Mathematics _ Practice Paper _ Algebra (part 1) | | | | | | | | | | | | | | | | | | | | | | | |
| Questions | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 13 | 14 | 15 | | | | | | | | | | | | |

In the longer term, use the new specification AS papers for additional revision for the 2017 specification and also the other websites we recommend. See the relevant sections on the Firefly page for further details.

What do I need to be able to do?

By the end of this chapter you should be able to:

- Solve quadratic equations using factorisation, the quadratic formula and completing the square
- Read and use $f(x)$ notation when working with functions
- Sketch the graph and find the turning point of a quadratic function
- Find and interpret the discriminant of a quadratic expression
- Use and apply models that involve quadratic functions

Solving quadratic equations

Remember that to solve a quadratic equation you should collect all the terms on one side so that the other side of the equation is 0.

When you solve the equation, if you have found the roots (i.e. where the graph of the quadratic function crosses the x -axis).

Factorising

Put the quadratic into brackets. If the product of two expressions is zero one or both of them must be equal to zero.

Eg Solve $x^2 + 6x + 8 = 0$
 $(x + 4)(x + 2) = 0$ We need two numbers that add to make the coefficient of x and multiply to give the constant term
 $x + 4 = 0$ or $x + 2 = 0$

Therefore: $x = -4$ or $x = -2$

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Eg Solve $3x^2 - 7x - 1 = 0$
 $a = 3$ $b = -7$ $c = -1$

Substitute into the formula:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times (3) \times (-1)}}{2 \times (3)}$$

Put each number in a bracket to avoid any sign errors

$$\text{Therefore: } x = \frac{7 + \sqrt{61}}{6} \text{ or } x = \frac{7 - \sqrt{61}}{6}$$

Make sure you give your answer in the form asked for. If they want exact leave in surd for like this. If they say 3sf or 1dp then make sure you give the decimal form of the answer

Y12 — Chapter 2 Quadratics

Key words:

- Quadratic — Where the highest exponent (index/power) of the variable is a square (2)
- Function — A special relationship where each input has a single output. It is often written as " $f(x)$ " where x is the input value
- Domain — All the values that go into a function
- Range — The set of all output values of a function
- Discriminant — The expression $b^2 - 4ac$ used when solving Quadratic Equations. It can "discriminate" between the possible types of answer

The general shape of a quadratic graph:

$$y = x^2$$



$$y = -x^2$$



Completing the square

Completing the square can be used to solve a quadratic equation but it is also very useful in determining the turning point of a quadratic function

The completed square form looks like this:

$$A(x + B)^2 + C = 0$$

Where the turning point is $(-B, C)$

Remember! If you need to solve the quadratic to find the roots and it is already in the completed square form, you don't need to factorise or use the formula you can just rearrange to find x .

The discriminant

The expression inside the square root sign is called the discriminant and tells you what type of roots to expect.

If $b^2 - 4ac > 0$ there are 2 real roots (i.e. the curve crosses the x -axis in 2 places)



If $b^2 - 4ac = 0$ there is 1 real root (i.e. the curve touches the x -axis in 1 place)



If $b^2 - 4ac < 0$ there are no real roots (i.e. the curve does not cross the x -axis)





Pure Mathematics PLC Checklist

| Chapter 2 | Quadratics | RAG |
|------------------|---|-----------|
| 2.1 | Solve quadratic equations by factorising. | |
| Exercise 2A | 1 2 3 4 5 | EX HW ASS |
| 2.1 | Solve quadratic equations using the quadratic formula. | |
| Exercise 2B | 1 2 3 4 | EX HW ASS |
| 2.2 | Complete the square for a quadratic function. | |
| Exercise 2C | 1 2 3 4 5 | EX HW ASS |
| 2.2 | Complete the square for a quadratic function and use to solve equations. | |
| Exercise 2D | 1 2 3 4 | EX HW ASS |
| 2.3 | Use function notation and find roots by solving $f(x) = 0$. | |
| Exercise 2E | 1 2 3 4 5 6 7 8 | EX HW ASS |
| 2.4 | Sketch quadratic graphs (detail roots, y-intercept, vertex and symmetry). | |
| Exercise 2F | 1 2 3 | EX HW ASS |
| 2.5 | Find the discriminant of a quadratic and determine the number of roots. | |
| Exercise 2G | 1 2 3 4 5 6 7 | EX HW ASS |
| 2.6 | Modelling with quadratics. | |
| Exercise 2H | 1 2 3 4 | EX HW ASS |
| Mixed Exercise 2 | Review questions from chapter (some may be set as part of homework). | |
| | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 | |

Independent Learning and Revision Opportunities

You should independently work on these questions over the year as ongoing revision.

Revision questions from the book covering the new specification.

| | |
|-----------|----------------------------------|
| Book | Review Exercise 1 (Pages 85-88). |
| Questions | 9 10 11 12 13 14 15 16 17 |

Examination questions on the topic from the old specification (paper pack on Firefly).

<https://chellaston.fireflycloud.net/maths-1/year-12/as-maths-revision/as-pure-revision-papers>

| | |
|---------------|--|
| Revision Pack | AS Mathematics _ Practice Paper _ Algebra (part 1) |
| Questions | 10 11 12 |

| | |
|---------------|--|
| Revision Pack | AS Mathematics _ Practice Paper _ Algebra (part 2) |
| Questions | 8 9 10 |

In the longer term, use the new specification AS papers for additional revision for the 2017 specification and also the other websites we recommend. See the relevant sections on the Firefly page for further details.

What do I need to be able to do?

By the end of this chapter you should be able to:

- Solve linear simultaneous equations using elimination or substitution
- Solve simultaneous equations: one linear and one quadratic
- Interpret algebraic solutions of equations graphically
- Solve linear and quadratic inequalities
- Interpret inequalities graphically
- Represent linear and quadratic inequalities graphically

Y12 – Chapter 3 Equations and inequalities

Key words:

- Simultaneous equations – Two or more equations that share variables
- Equation – a mathematical statement containing an equals sign, to show that two expressions are equal. An equation will have a finite set of solutions
- Inequality – An inequality compares two values, showing if one is less than, greater than, or simply not equal to another value

Solving simultaneous equations

| Method | Explanation | Works for |
|--------------|---|---|
| Elimination | Make the coefficients of one of the unknowns the same. (whichever seems easier) □ Add or subtract the equations to eliminate one unknown □ Solve the new equation to find the first unknown □ Substitute back into one of the original equations to find the other unknown. | Linear simultaneous equations |
| Substitution | Rearrange one of the equations (if necessary) to make either x or y the subject □ Substitute into the other equation □ Solve the new equation to find x or y . □ Substitute back into your rearranged equation to find the value of the other letter. *If after substituting you get a quadratic equation you can use the discriminant to determine the number of solutions | Linear only and one linear and one quadratic simultaneous equations |
| Graphically | On the same set of axes draw the graphs of both simultaneous equations The points of intersection will give you the solutions | Linear only and one linear and one quadratic simultaneous equations |

Linear inequalities

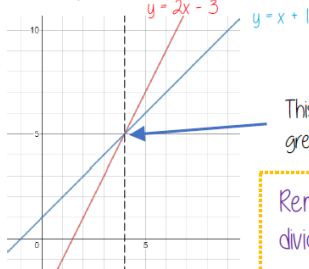
We solve linear inequalities the same way we would solve equations, except you get a range of solutions instead of one particular solution.

Eg Solve the inequality $2x - 3 > x + 1$ and sketch the outcome on a graph.

$$2x - 3 > x + 1$$

$$2x > x + 4$$

$$x > 4$$



This is the point where $2x-3$ becomes greater than $x+1$

Remember! If you multiply or divide an inequality by a negative number you have to reverse the inequality sign

Quadratic inequalities

To solve a quadratic inequality: always do a quick sketch (you will need to know the shape and the roots) then look for the appropriate part of the graph (ie. < 0 (below the x -axis) or > 0 (above the x -axis) depending on what you are looking for).

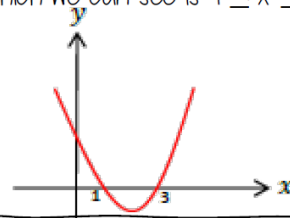
Eg Solve the inequality $x^2 + 4x + 3 \leq 0$

$$x^2 + 4x + 3 = 0$$

$$(x + 3)(x + 1) = 0$$

$$x = -3 \text{ or } x = -1 \quad \leftarrow \text{These are the roots}$$

We want the graph to be ≤ 0 so we want to describe the x values that represent the part of the curve under the x axis which we can see is $-3 \leq x \leq -1$





Pure Mathematics PLC Checklist

| Chapter 3 | Equations and Inequalities | RAG | | |
|------------------|---|-----|----|-----|
| 3.1 | Solve linear simultaneous equations by elimination or substitution. | | | |
| Exercise 3A | 1 2 3 4 5 | EX | HW | ASS |
| 3.2 | Solve simultaneous equations involving quadratic equations. | | | |
| Exercise 3B | 1 2 3 4 5 6 | EX | HW | ASS |
| 3.3 | Simultaneous equations and graphical representation (linear/quadratic). | | | |
| Exercise 3C | 1 2 3 4 5 6 7 8 9 | EX | HW | ASS |
| 3.4 | Solve linear inequalities. | | | |
| Exercise 3D | 1 2 3 | EX | HW | ASS |
| 3.5 | Solve quadratic inequalities. | | | |
| Exercise 3E | 1 2 3 4 5 6 7 8 9 | EX | HW | ASS |
| 3.6 | Interpret inequalities graphically. | | | |
| Exercise 3F | 1 2 3 | EX | HW | ASS |
| 3.7 | Represent inequalities as regions on graphs. | | | |
| Exercise 3G | 1 2 3 4 5 6 7 8 | EX | HW | ASS |
| Mixed Exercise 3 | Review questions from chapter (some may be set as part of homework). | | | |
| | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 | | | |

Independent Learning and Revision Opportunities

You should independently work on these questions over the year as ongoing revision.

Revision questions from the book covering the new specification.

| | |
|-----------|----------------------------------|
| Book | Review Exercise 1 (Pages 85-88). |
| Questions | 18 19 20 21 22 23 24 |

Examination questions on the topic from the old specification (paper pack on Firefly).

<https://chellaston.fireflycloud.net/maths-1/year-12/as-maths-revision/as-pure-revision-papers>

| | |
|---------------|--|
| Revision Pack | AS Mathematics _ Practice Paper _ Algebra (part 2) |
| Questions | 1 2 3 4 5 6 7 11 12 |

In the longer term, use the new specification AS papers for additional revision for the 2017 specification and also the other websites we recommend. See the relevant sections on the Firefly page for further details.

What do I need to be able to do?

By the end of this chapter you should be able to:

- Calculate the gradient of a line
- Understand the link between the equation of a line and its gradient and y-intercept
- Find the equation of a line
- Find the points of intersection of straight lines
- Know and use the rules for parallel and perpendicular gradients
- Solve length and area problems
- Use straight line graphs to construct mathematical models

Parallel or perpendicular?

Parallel lines — have the same gradient

Perpendicular lines — the product of the gradients is -1 (the gradients are negative reciprocals of each other)

Finding the distance between two point

Find the distance between (x_1, y_1) and (x_2, y_2) - Pythagoras' theorem

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Sketching a straight line

If you are given two points on the line, plot them and draw a line going through them

If you are given the equation in the form $y=mx+c$ plot the y intercept and then use the gradient to find additional points and join up

If you are given the equation in the form $ax+by+c=0$, find the x intercept (sub in $y=0$) and the y intercept ($x=0$), plot and join

Mathematical modelling

ALWAYS interpret your gradient and y intercept in the context of the question!

Y12 — Chapter 5 Straight line graphs

Key words:

- Gradient — How steep a line is
- Y-intercept — The point where a line or curve crosses the y-axis of a graph
- Parallel — Always the same distance apart and never touching
- Perpendicular — At right angles (90°) to
- Linear equation — An equation that makes a straight line when it is graphed

The equation of a straight line

There are several ways you can write an equation of a straight line:

| Form | Why it's useful |
|------------------------|--|
| $y=mx + c$ | The most commonly used form where m is the gradient and c the y-intercept |
| $y - y_1 = m(x - x_1)$ | When you have the gradient and a single point on the line; substitute them in for m , y_1 and x_1 - rearrange if necessary |
| $ax + by + c = 0$ | Useful when the gradient is a fraction and you want integer values |

Finding the gradient of a straight line

The gradient (m) of the line that joins the points (x_1, y_1) and (x_2, y_2) use the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Finding the point of intersection

Use simultaneous equations either by elimination or substitution



Pure Mathematics PLC Checklist

| Chapter 5 | Straight Line Graphs | RAG | | |
|------------------|---|-----|----|-----|
| 5.1 | Calculate the gradient between two points. | | | |
| Exercise 5A | 1 2 3 4 5 6 7 8 9 10 | EX | HW | ASS |
| 5.1 | Use the equation of a line to find the gradient and intercepts with axes. | | | |
| Exercise 5B | 1 2 3 4 5 6 7 8 9 10 11 12 | EX | HW | ASS |
| 5.2 | Find the equation of a line using an algebraic method. | | | |
| Exercise 5C | 1 2 3 4 5 6 | EX | HW | ASS |
| 5.2 | Find the equation of a line and intersections with other lines or axes. | | | |
| Exercise 5D | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 | EX | HW | ASS |
| 5.3 | Know and use facts involving gradients of parallel lines. | | | |
| Exercise 5E | 1 2 3 4 5 6 7 8 | EX | HW | ASS |
| 5.3 | Know and use facts involving gradients of perpendicular lines. | | | |
| Exercise 5F | 1 2 3 4 5 6 7 8 9 10 11 12 | EX | HW | ASS |
| 5.4 | Find the length of a line segment and calculate areas. | | | |
| Exercise 5G | 1 2 3 4 5 6 7 8 9 10 11 12 | EX | HW | ASS |
| 5.5 | Model contextual situations using straight line graphs. | | | |
| Exercise 5H | 1 2 3 4 5 6 7 8 9 | EX | HW | ASS |
| Mixed Exercise 5 | Review questions from chapter (some may be set as part of homework). | | | |
| | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 | | | |

Independent Learning and Revision Opportunities

You should independently work on these questions over the year as ongoing revision.

Revision questions from the book covering the new specification.

| | | | | | | | | | | | | | | | | | |
|-----------|------------------------------------|---|---|---|---|---|---|--|--|--|--|--|--|--|--|--|--|
| Book | Review Exercise 2 (Pages 226-229). | | | | | | | | | | | | | | | | |
| Questions | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | | | | | | | | | |

Examination questions on the topic from the old specification (paper pack on Firefly).
<https://chellaston.fireflycloud.net/maths-1/year-12/as-maths-revision/as-pure-revision-papers>

| | | | | | | | | | | | | | | | | | |
|---------------|---|---|---|---|---|---|--|--|--|--|--|--|--|--|--|--|--|
| Revision Pack | AS Mathematics _ Practice Paper _ Coordinate geometry | | | | | | | | | | | | | | | | |
| Questions | 1 | 2 | 3 | 4 | 5 | 6 | | | | | | | | | | | |

In the longer term, use the new specification AS papers for additional revision for the 2017 specification and also the other websites we recommend. See the relevant sections on the Firefly page for further details.

What do I need to be able to do?

By the end of this chapter you should be able to:

- Cancel factors in algebraic fractions
- Divide a polynomial by a linear factor
- Use the factor theorem to factorise a cubic expression
- Construct mathematical proofs using algebra
- Use proof by exhaustion and disproof by counter example

Algebraic fractions

Algebraic fractions behave, and follow the same rules as numerical fractions.

When simplifying algebraic fractions, where possible factorise the numerator and denominator and then cancel out common factors

Eg Simplify

$$\frac{2x^2 + 11x + 12}{x^2 + 7x + 12}$$

$2x^2 + 11x + 12$ factorises to $(2x + 3)(x + 4)$

$x^2 + 7x + 12$ factorises to $(x + 3)(x + 4)$

So, the fraction can be written as:

$$\frac{(2x + 3)(x + 4)}{(x + 3)(x + 4)}$$

(x+4) is the common factor so it cancels

Proof

In a mathematical proof you must:

- State any information or assumptions you are using
- Show every step clearly
- Each step should follow logically from the previous step
- Make sure you have covered all possible cases
- Write a statement of proof at the end of your working

To prove an identity you should:

- Start with one side of the identity
- Manipulate it to match the other side
- Show every step of your working

Y12 – Chapter 7 Algebraic Methods

Key words:

- Polynomial – A polynomial can have constants, variables (and exponents that can be combined using addition, subtraction, multiplication and division, but:
 - no division by a variable.
 - a variable's exponents can only be 0 or a positive integer.
 - not an infinite number of terms.
- Proof – Logical mathematical arguments used to show the truth of a mathematical statement.

In a proof we can use:

- axioms (self-evident truths) such as "we can join any two points with a straight-line segment" (one of Euclid's Axioms)
- existing theorems, that have themselves been proven.

Polynomial division

$$(6x^3 + 28x^2 - 7x + 15) \div (x + 5)$$

Method 1 – Long division

Divide the first term of the polynomial by x ($6x^3 \div x = 6x^2$)
 Multiply $(x+5)$ by $6x^2$ and write under polynomial
 Subtract and bring down $-7x$

Repeat for each term of the polynomial

Method 2 – Box method

$$\begin{array}{r} 6x^3 + 28x^2 - 7x + 15 \div x + 5 \\ \hline \begin{array}{|c|c|c|} \hline 6x^2 & -2x & 3 \\ \hline \end{array} \\ \begin{array}{|c|c|c|} \hline x & 6x^3 & -2x^2 & 3x \\ \hline \end{array} \\ \begin{array}{|c|c|c|} \hline +5 & 30x^2 & -10x & 15 \\ \hline \end{array} \end{array}$$

Divide the first term of the polynomial by x ($6x^3 \div x = 6x^2$)
 Multiply $+5$ by $6x^2$ and write in box ($30x^2$)
 Subtract $30x^2$ from x^2 term in the polynomial and complete box ($28x^2 - 30x^2 = -2x^2$) and write in box
 Divide $-2x^2$ by x ($-2x$)
 Multiply $+5$ by $-2x$ and write in box ($-10x$)
 Subtract $-10x$ from the x term in the polynomial ($-7x - -10x = 3x$) and write in box
 Divide $3x$ by x (3)
 Multiply $+5$ by 3 and write in box (15)
 If you collect the terms in your boxes it should match your polynomial

Proof continued...

Proof by exhaustion – break the statement into smaller cases and prove each one separately

Proof by counter example – give one example that does not work

Factor Theorem

If $f(p) = 0$, then $(x - p)$ is a factor of $f(x)$

If $(x - p)$ is a factor of $f(x)$, then $f(p) = 0$



Pure Mathematics PLC Checklist

| Chapter 7 | Algebraic Methods | RAG | | |
|------------------|---|-----|----|-----|
| 7.1 | Simplify algebraic fractions. | | | |
| Exercise 7A | 1 2 3 | EX | HW | ASS |
| 7.2 | Divide a polynomial into a linear factor and use to factorise. | | | |
| 7.2 | Use the factor and remainder theorems for linear divisors. | | | |
| Exercise 7B | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 | EX | HW | ASS |
| 7.3 | Use the factor theorem to identify a factor and fully factorise cubics. | | | |
| Exercise 7C | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 | EX | HW | ASS |
| 7.4 | Prove mathematical statements using proof by deduction. | | | |
| Exercise 7D | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 | EX | HW | ASS |
| 7.5 | Use other methods of proof: exhaustion and counter example. | | | |
| Exercise 7E | 1 2 3 4 5 6 7 8 9 10 | EX | HW | ASS |
| Mixed Exercise 7 | Review questions from chapter (some may be set as part of homework). | | | |
| | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 | | | |

Independent Learning and Revision Opportunities

You should independently work on these questions over the year as ongoing revision.

Revision questions from the book covering the new specification.

| | | | | | | | | | | | | | | | | | | | | | | | | |
|-----------|------------------------------------|----|----|----|----|----|----|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| Book | Review Exercise 2 (Pages 226-229). | | | | | | | | | | | | | | | | | | | | | | | |
| Questions | 14 | 15 | 16 | 17 | 18 | 19 | 20 | | | | | | | | | | | | | | | | | |

Examination questions on the topic from the old specification (paper pack on Firefly).

<https://chellaston.fireflycloud.net/maths-1/year-12/as-maths-revision/as-pure-revision-papers>

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---------------|--|----|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| Revision Pack | AS Mathematics _ Practice Paper _ Algebra (part 1) | | | | | | | | | | | | | | | | | | | | | | | |
| Questions | 16 | 17 | | | | | | | | | | | | | | | | | | | | | | |

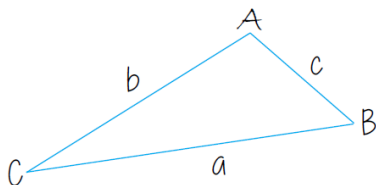
In the longer term, use the new specification AS papers for additional revision for the 2017 specification and also the other websites we recommend. See the relevant sections on the Firefly page for further details.

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use the cosine rule to find a missing side or angle
- Use the sine rule to find a missing side or angle
- Find the area of a triangle using an appropriate formula
- Solve problems involving triangles
- Sketch the graphs of the sine, cosine and tangent functions
- Sketch simple transformations of these graphs

The Cosine Rule



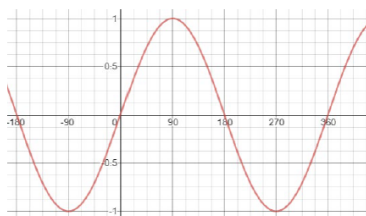
To find a missing side: $a^2 = b^2 + c^2 - 2bc \cos A$

To find a missing angle: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Use the cosine rule when you either:

- Know two sides and the angle between them and want to know the third side
- Know three sides and want to find an angle

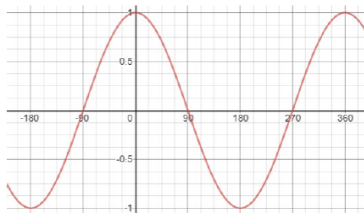
Graphs of sine, cosine and tangent



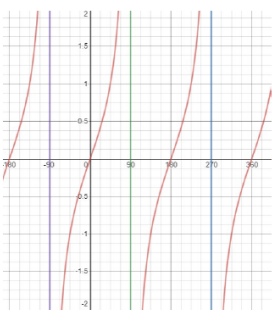
The graph of $y = \sin \theta$
Repeats every 360°
Crosses the x axis every 180°
Has a maximum value of 1 and a minimum value of -1

The graph of $y = \cos \theta$
Repeats every 360°
Crosses the x axis at $-90^\circ, 90^\circ, 270^\circ \dots$

Has a maximum value of 1 and a minimum value of -1



The graph of $y = \tan \theta$
Repeats every 180°
Crosses the x axis at $-180^\circ, 0, 180^\circ, 360^\circ \dots$
Has vertical asymptotes at $x = -90^\circ, x = 90^\circ, x = 270^\circ \dots$

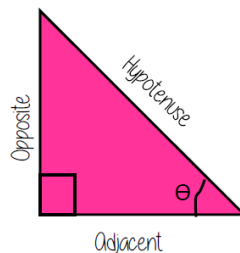


Y12 - Chapter 9 Trigonometric Ratios

Key words:

- Periodic function - A function (like Sine and Cosine) that repeats forever

Sine, Cosine and Tangent



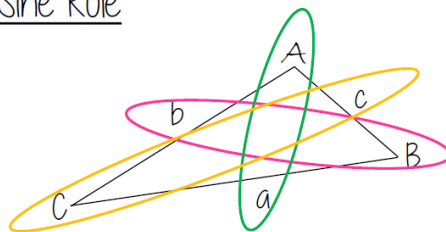
$\sin \theta = \text{Opposite} / \text{Hypotenuse}$

$\cos \theta = \text{Adjacent} / \text{Hypotenuse}$

$\tan \theta = \text{Opposite} / \text{Adjacent}$

Pure
Maths
Year
1/AS

The Sine Rule



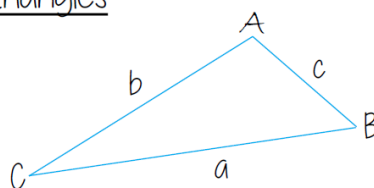
To find a missing side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

To find a missing angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Use the sine rule when you have opposite pairs of angles and sides

The sine rule sometimes produces two possible solutions for a missing angle: $\sin \theta = \sin(180 - \theta)$

Areas of triangles



$$\text{Area} = \frac{1}{2} ab \sin C$$



Pure Mathematics PLC Checklist

| Chapter 9 | Trigonometric Ratios | RAG | | |
|------------------|--|-----|----|-----|
| 9.1 | Use the cosine rule to find missing sides and angles in triangles. | | | |
| Exercise 9A | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 | EX | HW | ASS |
| 9.2 | Use the sine rule to find missing sides and angles in triangles. | | | |
| Exercise 9B | 1 2 3 4 5 6 7 8 9 10 11 12 | EX | HW | ASS |
| 9.2 | The ambiguous case of the sine rule. | | | |
| Exercise 9C | 1 2 3 4 5 6 | EX | HW | ASS |
| 9.3 | Areas of triangles. | | | |
| Exercise 9D | 1 2 3 4 5 6 | EX | HW | ASS |
| 9.4 | Solving triangle problems. | | | |
| Exercise 9E | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 | EX | HW | ASS |
| 9.5 | Sketch graphs of trigonometric functions: sin/cos/tan. | | | |
| Exercise 9F | 1 2 3 4 | EX | HW | ASS |
| 9.6 | Transformations graphs of trigonometric functions. | | | |
| Exercise 9G | 1 2 3 4 5 6 7 8 9 | EX | HW | ASS |
| Mixed Exercise 9 | Review questions from chapter (some may be set as part of homework). | | | |
| | 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 | | | |

Independent Learning and Revision Opportunities

You should independently work on these questions over the year as ongoing revision.

Revision questions from the book covering the new specification.

| | | | | | | | | | | | | | | | | |
|-----------|------------------------------------|----|----|----|----|----|----|----|----|--|--|--|--|--|--|--|
| Book | Review Exercise 2 (Pages 226-229). | | | | | | | | | | | | | | | |
| Questions | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | | | | | | | |

No examination questions in year 12 topic packs.

Sine and cosine rule does appear in the year 13 packs in the radians section.

Focus on the review exercise from the book for this topic.

In the longer term, use the new specification AS papers for additional revision for the 2017 specification and also the other websites we recommend. See the relevant sections on the Firefly page for further details.